Lower Bounds for Park and Loop Delivery Problems

Claudia Bode

Chair of Logistics Management, Gutenberg School of Management and Economics, Johannes Gutenberg University Mainz, Jakob-Welder-Weg 9, D-55128 Mainz, Germany.

Abstract

Combining the arc routing problem with the problem of determining locations for depots results in location arc routing problems (LARP). Several ways of connecting depots and assigning routes to the depots are presented in the literature. This paper addresses the different types of LARP and classifies the literature according to these types. Park and loop is one of them, where routes have to be assigned to the depots in such a way that the start and end point of the routes is the same depot. Extending this problem by also allowing a service to be performed with a large transfer vehicle that connects the depots is called park and loop with curbline routes. For these two types of LARP, four mixed integer formulations are presented. The two models, each representing one of the types, differ in how they formulate feasible routes connecting the depots. While the first model uses generalized subtour elimination constraints, the second one uses flow variables. The quality of the formulations is tested in a computational study.

Key words: park and loop, location arc routing problem

1. Introduction

Since their introduction by Golden and Wong (1981), (mixed) capacitated arc routing problems ((M)CARP) have received increasing attention over the last decades. An optimal CARP solution consists of a set of cost minimal feasible routes for a fleet of vehicles such that every edge with positive demand is serviced by exactly one vehicle and the capacity of each vehicle is met. Each route starts and ends at a specific depot node. In the mixed case, the underlying network is represented as a mixed graph in order to achieve more realistic models of real world problems (Belenguer *et al.*, 2006). Because for the CARP a fixed depot location is required, some authors relax this assumption, which leads to a location arc routing problem (LARP) (Levy and Bodin, 1989). This problem consists of simultaneously determining locations of depots and obtaining routes connected to the depots. Unfortunately, the term LARP is used by several authors even though the ways of connecting routes to the depots and connecting the depots to each other differ.

Therefore, the first contribution of this paper is to classify the literature into the categories defined by Bodin and Levy (2000) for mail delivery application. The second contribution is to present a total of four mathematical formulations for the park and loop (PAL) problem, which is a variant of the LARP, and the combined park and loop with curbline routes (CurbPAL) problem, which is an extension of the PAL problem. Characteristic of the PAL problem is the combination of the routing of a transfer vehicle that takes the postman from the depot to parking lots and the routing of smaller delivery tours starting and ending at each parking lot. In the PAL problem, service is only allowed within routes starting at the parking lots. An extended variant - the CurbPAL problem - can be formulated to also allow service while the transfer vehicle connecting the parking lots is being driven. So far, the formulations presented in the literature either (*i*) do not describe the connections between the parking lots or (*ii*) do not have the delivery routes starting and ending at the same parking lots. In this work, these points are integrated into a mathematical formulation

Email address: `claudia.bode@uni-mainz.de` (Claudia Bode)

for the first time. Therefore, two models for each problem (PAL and CurbPAL), based on flow formulations of Gouveia *et al.* (2010) and Hashemi Doulabi and Seifi (2013) are presented. These two models differ in the way they handle infeasible subtours of the transfer vehicle. The first model forbids such subtours by subsequently adding generalized subtour elimination constraints; the second model uses additional flow variables to ensure connectivity.

The rest of the paper is structured as follows: The description of different types of LARP and their classification into the categories of Bodin and Levy (2000) are presented in Section 2. The four models (two each for the PAL and the extended PAL problem) are introduced in Section 3. In Section 4, computational results for the presented models are shown, and a conclusion is drawn in Section 5.

2. Classification of location arc routing problems

It is well known that the locations of depots have an influence on the costs of routes assigned to them (Salhi and Rand, 1989). Therefore, it is advantageous to simultaneously solve the problem of locating the depots and the routing problem, which is then called a location routing problem (LRP). The question is how to locate depots and assign routes to those depots in order to minimize the overall costs, which consist of depot and vehicle costs. The types of routes can be further divided into routes requiring a service at nodes (still called LRP) and routes performing a service along edges or arcs. The latter version is called a location arc routing problem (LARP).

While the LRP is addressed extensively, only a few articles target the arc routing case. A survey on applications for mail delivery, winter gritting, and garbage collection problems can be found in (Assad and Golden, 1995). The first two papers studying the LARP were in the context of mail delivery for the Canada Post Corporation (Roy and Rousseau, 1989) and the United States Postal Service (Levy and Bodin, 1989). Later, Assad and Golden (1995) and Bodin and Levy (2000) observed that different ways of designing delivery routes exist, and hence of assigning routes to the located depots. In the following we will present delivery types suggested by Bodin and Levy (2000) and classify articles on the LARP into these types.

1. When delivering mail in a curbline route, the postman starts at the depot and drives along streets where mail has to be delivered. He can fill the mailboxes directly from his vehicle. The working time of the postman is completed, when he returns to the depot. A similar mode is the dismount route, but in contrast to the curbline mode, more than 50% of the mail is delivered directly to the door, such that the postman has to leave his vehicle. No parking lots have to be opened and only one type of vehicle is involved. This delivery mode can be modeled as a standard CARP, without the inclusion of any location aspects. Several exact and heuristic methods have been presented for this kind of problem and are surveyed in (Belenguer *et al.*, 2013) and (Prins, 2013).
2. The second delivery mode is called relay box routes, where a postman starts the delivery route with a full handcart at some point. He delivers mail until his handcart is empty, then walks to a relay box to refill his handcart and to continue his route. Subsequent refillings can be done at the same or at a different relay box. There are no constraints on where the last delivery ends. The transfer from the depot to the starting point and from the end point back to the depot is done by some means of transportation determined by the postal service. Disregarding this transfer transport, only one type of vehicle is involved. The main aspect, as stated in (Roy and Rousseau, 1989), is that of tour length restrictions for the morning and afternoon tours, which can be seen as capacity restrictions of a specific tour (Assad and Golden, 1995). Additionally, Bodin and Levy (2000) mention that the mail delivery from the depot to the relay boxes is not considered simultaneously, but can be modeled as a node routing problem with time windows. However, combining these two aspects without time windows and assuming that the locations of the relay boxes are not given in advance, the problem is to find these locations in order to meet the load capacity restrictions of the postman and to obtain his walking route and the filling route of the relay boxes simultaneously. Such problems can be modeled as LARPs.

   There are some papers that can be classified in this problem category: Ghiani *et al.* (2001) consider the CARP with intermediate facilities. Applications are waste management problems, where one vehicle collects garbage and whenever the vehicle capacity is met, it has to be unloaded at a dump site or incinerator. Another application is road gritting, where intermediate facilities represent located boxes of sand or chemicals. What these problems have in common is that a vehicle starts at the depot collecting or delivering goods and has to be unloaded or replenished at the intermediate facilities, which represent relay boxes. After servicing all required edges, it returns to the depot. However, no filling routes of the relay boxes are considered. Amaya *et al.* (2007, 2010) consider the mail delivery to the relay boxes and the route for the postman simultaneously. They look at a road marking problem where a fleet of vehicles starts at the depot, marks required roads and returns to the depot. A second type of vehicle also starts at the depot but traverses to specific positions (the relay boxes) to replenish the first vehicle type with color for marking the roads. Salazar-Aguilar *et al.* (2013) extend this problem to a synchronized arc and node routing problem by including time aspects. They consider the locations of refilling and the routes for refilling and for the marking vehicles simultaneously.

3. A delivery mode very similar to the relay box routes is the park and loop mode. There, several types of vehicles for traversing streets and mail delivery are available. In the standard problem, as proposed in (Bodin and Levy, 2000), a park and loop route consists of driving a transfer vehicle from the depot to a parking lot, getting off and walking to deliver mail at a set of street segments and returning to the parking lot. In the following, these loops are referred to as walking loops. Further, either the handcart is filled again with mail and a new walking loop starts, or the postman drives the vehicle to the next parking lot. When all mail has been distributed, the postman drives back to the depot. Possible extensions can be the use of several vehicle types for mail delivery within the walking loops. The main difference to the relay box mode is that the postman has to bring all mail for delivery by himself and he has to return to the parking lot at the end of each delivery route. Determining the locations of parking lots and obtaining the corresponding walking loops can be modeled as an LARP. Levy and Bodin (1989) considered the delivery problem of the United States Postal Service, where each delivery loop has to start and finish at the same depot. The presented heuristic follows the location-allocation-routing principle, where the location of depots and the allocation of street segments to a depot are evaluated according to small deadheading time, balanced workload of each partition, and minimal number of located depots. Amberg *et al.* (2000) transformed the problem into an arc-constraint minimum spanning tree with multiple centers and developed a tabu search algorithm to solve the problem. In contrast to the former approach, a heterogeneous fleet of vehicles is available to deliver mail. Several objective functions, such as minimizing the deadheading time, balancing the workload, and accounting for customer priority, are evaluated. Recently, Hashemi Doulabi and Seifi (2013) presented a flow formulation of the multi depot problem with homogeneous vehicles, where a predefined number of depots can be opened. The objective function takes into account service and deadheading costs, hiring costs for each vehicle in use, and dumping costs for open depots. To analyze the performance of their simulated annealing approach, a lower bounding model was presented.
   What is common to all approaches considered so far are the missing interconnections of the depots by a transfer vehicle, which is part of the delivery mode proposed by Bodin and Levy (2000). Nevertheless, these approaches can be seen as LARPs with park and loop characteristics, because all vehicles of the walking loops have to return to their starting depots.
4. The last delivery mode that Bodin and Levy (2000) mentioned is a combination of park and loop routes and curbline routes. There, the postman is able to deliver mail both while driving the transfer vehicle to a parking lot or while performing a walking loop starting and ending at a parking lot. So far, no model has been presented in the literature that considers this kind of problem. As park and loop routes are part of the problem, again, an LARP model appears to be convenient.
5. The combination of relay box routes and curbline routes is another possible delivery mode. Del Pia and Filippi (2006) consider a garbage collection problem with two types of collection vehicles. The capacity and the underlying street network differ for the smaller satellite vehicles and the larger compactor trucks. Both types of vehicles are able to collect garbage and whenever satellites and trucks meet at the same node at the same time, the smaller vehicle can dump its load into the container of the larger truck. The problem considered is that of simultaneously determining the dump locations and the routes for each vehicle.

3. Mail delivery routes with park and loop characteristics

As pointed out in the previous section, no formulation representing the full problem specifications defined by Bodin and Levy (2000) for both the pure park and loop delivery mode and the combination with curbline delivery exists. Especially the aspect of connecting the parking lots by a transfer vehicle is omitted. This section will introduce four mixed integer programs, two modeling the park and loop mode and two formulations for the combined park and loop with curbline mode. The presented models are based on the multi-depot LARP defined in (Hashemi Doulabi and Seifi, 2013). They extended the flow formulation of the single-depot MCARP of Gouveia *et al.* (2010). However, the open parking lots are not connected in their approach. The formulations shown below extend their model in order to integrate a transfer route starting and ending at a specific loop-depot and visiting all open parking lots. For the first two formulations, the postman is not allowed to perform any service on edges or arcs while driving the transfer vehicle. In order to model combined park and loop and curbline routes, a service is also allowed with the transfer vehicle for the second two models by further modifications.

Some notation valid for all mathematical formulations is presented first. The (combined) park and loop model considered in this paper is defined on a mixed graph $G = (V, E \cup A')$ with node set V, edge set E, and arc set A'. With each edge or arc (i, j), a non-negative demand $q_{ij} \geq 0$ is associated. The set of required edges or arcs, i.e., $q_{ij} > 0$, is indicated by a subscript $\cdot_R$. A subset of nodes $D \subseteq V$ defines the possible locations of parking lots. The maximum number of open parking lots is D_{max}. Whenever a parking lot $d \in D$ is open, costs OC_d occur. At each parking lot $d \in D$, a heterogeneous fleet K of $|K|$ different vehicles with capacity Q^k $k \in K$ is available to perform the service. Each vehicle k selected for servicing creates hiring costs λ^k. The transfer vehicle is an additional vehicle indexed by $k = 0$ and is stationed at a specific loop-depot $d_L \in D$. Whenever traversing an edge or arc (i, j) with vehicle $k \in K \cup \{0\}$, traversing costs c_{ij}^k occur. Service costs $c_{ij}^{serv,k}$ are associated with required edges or arcs (i, j) when the service is performed with vehicle $k \in K$. If it is also allowed to perform a service with the transfer vehicle, service costs $c_{ij}^{serv,0}$ occur.

Some modifications on the graph are made, as the underlying formulation of the multi-depot LARP is based on flow variables. Each edge $\{i, j\}$ is replaced by two opposite arcs (i, j) and (j, i). Therefore, the directed graph $G = (V, A'')$ consists of the node set V and the new arc set $A'' = A' \cup \{(i, j), (i, j) | \{i, j\} \in E\}$. To be able to use a parking lot also as an intermediate node of a walking loop, each node that represents a potential parking lot is duplicated and the set of duplicated parking lots is denoted by D'. New non-required arcs (d, d') and (d', d) with zero deadheading costs are added. The final graph representing the underlying network is then given by $G = (V \cup D', A)$ with $A = A'' \cup \{(d, d'), (d', d) | d \in D\}$.

Throughout the paper we will use the following standard notation. For a set $S \subseteq V$, we denote by $\delta(S)$ the set of edges with exactly one endpoint in S and by $\delta^+(S)$ and $\delta^-(S)$ the cut set of arcs leaving and entering set S, respectively. Similarly to the notation of required edges and arcs, we denote by $\delta_R(S) = E_R \cap \delta(S)$ and $\delta_R^*(S) = A_R \cap \delta^*(S)$ with $* \in \{+, -\}$ the set of required edges or arcs in the cut. To further shorten the notation, we will write $\delta(i)$ ($\delta^*(i)$ with $* \in \{+, -\}$) instead of $\delta(\{i\})$ ($\delta^*(\{i\})$ with $* \in \{+, -\}$).

3.1. Park and loop models

A park and loop solution consists of a set of open parking lots, a transfer route connecting these parking lots and a set of tours starting and ending at each parking lot, while respecting the capacity restriction of each vehicle. Inspired by the simple cycle problem (Fischetti *et al.*, 2004) and the directed version of the prize collecting traveling salesman problem (Balas, 1989), the first of our models uses standard generalized subtour elimination constraints (SEC) for connecting the parking lots. Cutting plane procedures to identify violated generalized SEC can easily be adapted to the park and loop case. However, because of the exponential number of constraints, the problem cannot be stated at once. The second of our models uses a flow formulation to connect the open parking lots, similarly to the single depot flow formulation of Gavish and Graves (1978) for the traveling salesman problem. The advantage of this formulation is the linear number of constraints needed to eliminate subtours. However, additional flow variables have to be introduced for the formulation.

3.1.1. Park and loop model with generalized SEC

To give a mathematical formulation, some further notation is needed: Let SP_{ij} be the cost of the shortest deadheading path between i and j for the transfer vehicle, i.e., the costs of the shortest path are determined by summing up the corresponding traversing costs. Variables $x_{ij}^{k,l}$ take value 1 if arc (i,j) is serviced by vehicle k that is linked to a parking lot by label l, and 0 otherwise. Variables $y_{ij}^{k,l}$ count the number of times an arc (i,j) is traversed by vehicle k linked to a parking lot by l. In order to calculate the hiring cost of vehicles that perform a service, variables DC_d are introduced. For each potential location of a parking lot a variable z_d exists and takes value 1 if the parking lot is used, and 0 otherwise. The tour of a vehicle k is linked to a parking lot by variables T_d^l. These variables take value 1 if node d is selected as a parking lot and label l is assigned to this place. Flow variables $f_{ij}^{k,l}$ for every arc (i,j) are used to ensure the connectivity of the tour of vehicle k linked to a parking lot by l within the walking loops. Finally, variables r_{ij} define the route of the transfer vehicle and take value 1 if parking lot i is connected with parking lot j. The (PAL) model then reads:

$$\text{(PAL)}\quad \min \sum_{l=1,\dots,D_{max}} \left(\sum_{k\in K} \left(\sum_{(i,j)\in A_R} c_{ij}^{serv,k} x_{ij}^{k,l} + \sum_{(i,j)\in A} c_{ij}^{k} y_{ij}^{k,l} \right) \right) + \sum_{i\in D'} \sum_{j\in D'\setminus\{i\}} SP_{ij} r_{ij} + \sum_{d'\in D'} (DC_{d'} + OC_{d'} z_{d'}) \tag{1}$$

$$\text{s.t.} \sum_{j\in\delta^+(i)} y_{ij}^{k,l} + \sum_{j\in\delta_R^+(i)} x_{ij}^{k,l} = \sum_{j\in\delta^-(i)} y_{ji}^{k,l} + \sum_{j\in\delta_R^-(i)} x_{ji}^{k,l} \quad \forall i\in V,\quad \forall k\in K,\quad l=1,\dots,D_{max} \tag{2}$$

$$\sum_{l=1,\dots,D_{max}} \sum_{k\in K} x_{ij}^{k,l} = 1 \quad \forall (i,j)\in A'_R, \qquad \sum_{l=1,\dots,D_{max}} \sum_{k\in K} (x_{ij}^{k,l} + x_{ji}^{k,l}) = 1 \quad \forall \{i,j\}\in E_R \tag{3}$$

$$y_{d'd}^{k,l} \le T_{d'}^{l} \qquad \forall d'\in D',\quad \forall k\in K,\quad l=1,\dots,D_{max} \tag{4}$$

$$\sum_{j\in\delta^-(i)} f_{ji}^{k,l} - \sum_{j\in\delta^+(i)} f_{ij}^{k,l} = \sum_{j\in\delta_R^-(i)} q_{ji} x_{ji}^{k,l} \qquad \forall i\in V,\quad \forall k\in K,\quad l=1,\dots,D_{max} \tag{5}$$

$$\sum_{(i,j)\in A_R} q_{ij} x_{ij}^{k,l} - M(1-T_d^l) \le f_{d'd}^{k,l} \le \sum_{(i,j)\in A_R} q_{ij} x_{ij}^{k,l} + M(1-T_d^l) \qquad \forall d'\in D',\quad \forall k\in K,\quad l=1,\dots,D_{max} \tag{6}$$

$$f_{d'd}^{k,l} \le M T_d^l \qquad \forall d'\in D',\quad \forall k\in K,\quad l=1,\dots,D_{max} \tag{7}$$

$$f_{dd'}^{k,l} = 0 \qquad \forall d'\in D',\quad \forall k\in K,\quad l=1,\dots,D_{max} \tag{8}$$

$$f_{ij}^{k,l} \le Q^k (x_{ij}^{k,l} + y_{ij}^{k,l}) \qquad \forall (i,j)\in A,\quad \forall k\in K,\quad l=1,\dots,D_{max} \tag{9}$$

$$DC_{d'} \ge \lambda\Big(\sum_{k\in K} y_{d'd}^{k,l}\Big) - M(1-T_{d'}^l) \qquad \forall d'\in D',\quad l=1,\dots,D_{max} \tag{10}$$

$$\sum_{d'\in D'} z_{d'} \le D_{max} \tag{11}$$

$$\sum_{l=1,\dots,D_{max}} T_{d'}^l = z_{d'} \qquad \forall d'\in D' \tag{12}$$

$$\sum_{d'\in D'} T_{d'}^l \le 1 \qquad l=1,\dots,D_{max} \tag{13}$$

$$x_{ij}^{k,l} \le \sum_{d'\in D'} T_{d'}^l \qquad \forall (i,j)\in A_R,\quad \forall k\in K,\quad l=1,\dots,D_{max} \tag{14}$$

$$y_{ij}^{k,l} \le M \sum_{d'\in D'} T_{d'}^l \qquad \forall (i,j)\in A,\quad \forall k\in K,\quad l=1,\dots,D_{max} \tag{15}$$

$$\sum_{j \in D' \setminus \{i\}} r_{ij} = z_i \quad \forall i \in D' \setminus \{d_L\}, \qquad \sum_{i \in D' \setminus \{j\}} r_{ij} = z_j \quad \forall j \in D' \setminus \{d_L\} \tag{16}$$

$$\sum_{i \in S} \sum_{j \notin S} r_{ij} \geq z_h \quad \forall S \subseteq D', \, d_L \in S, \, h \in D' \setminus S \tag{17}$$

$$x^{k,l} \in \{0,1\}^{|A_R|}, \quad y^{k,l} \in \mathbb{Z}_+^{|A|}, \quad f_{ij}^{k,l} \geq 0 \quad \forall (i,j) \in A, \quad \forall k \in K, \quad l = 1, \ldots, D_{max}$$

$$z \in \{0,1\}^{|D'|}, \quad T^l \in \{0,1\}^{|D'|} \quad l = 1, \ldots, D_{max}, \quad r_{ij} \in \{0,1\} \quad \forall i,j \in D'. \tag{18}$$

The objective (1) seeks to minimize the sum over service and deadheading costs of the walking loops, the connection of parking lots by the transfer vehicle, and the costs for opening a depot and hiring a vehicle. Constraints (2)–(15) define feasible routes of the service vehicles $k \in K$, while constraints (16) and (17) ensure a feasible transfer tour. The connectivity of service vehicles at each node is ensured by equations (2) and the service of each required arc and edge by exactly one vehicle is guaranteed by equations (3). Inequalities (4) ensure a maximum of one traversal from the dummy parking lot to the original one if d is an open parking lot. Constraints (5)–(8) are flow conservation constraints. Together with the coupling constraint (9) they ensure the elimination of infeasible subtours.

- Equations (5) ensure that the difference between inflow and outflow of node i is exactly the demand delivered on arcs entering node i. This type of constraint is also known as a generalized flow conservation constraint.
- Inequalities (6) ensure that if the parking lot d is open, the flow leaving this node by vehicle k is equal to the demand delivered by the same vehicle linked to the parking lot by l.
- Inequalities (7) ensure that the outgoing flow from parking lot d is zero if d is not open.
- Equations (8) ensure that there is no flow left when finishing the tour by returning to the dummy parking lot.
- Inequalities (9) are upper bounds on the flow of an arc (i, j) and couple flow variables with traversing and servicing variables.

Next, inequalities (10) calculate the total hiring costs of tours assigned to parking lot d. Constraints (11)–(15) restrict the number of open parking lots, the number of labels assigned to a parking lot and the vehicles assigned to a parking lot by a label l. More precisely:

- Inequality (11) ensures at maximum D_{max} parking lots are open.
- Equalities (12) link the variables T_d^l to a specific parking lot d.
- Inequalities (13) ensure that no more than one depot d is linked to a label l.
- Inequalities (14) and (15) ensure that a tour of vehicle k linked to parking lot d by l is constructed only if there is a link between d and l.

The last two sets of restrictions define a feasible tour for the transfer vehicle. Equalities (16) are generalized node degree constraints and state that exactly one arc has to leave (enter) node i if i is selected as a parking lot. If d_L is the only open parking lot, the transfer vehicle does not need to connect d_L to other parking lots. Therefore, no such constraint exists for the loop-depot d_L. Constraints (17) eliminate subtours of the transfer vehicle including an open parking lot h disconnected from the loop-depot d_L. This version of the generalized subtour elimination constraints stated here assumes that the loop-depot d_L is always a possible parking lot.

Additional improving constraints, as presented in (Hashemi Doulabi and Seifi, 2013) and (Gouveia *et al.*, 2010), are added:

$$f_{ij}^{kl} \leq q_{ij} x_{ij}^{kl} \qquad \forall (i,j) \in A_R, \quad \forall k \in K, \quad l = 1, \ldots, D_{max} \tag{19}$$

$$f_{ij}^{kl} \leq \left(y_{ij}^{kl} - 1\right) \min_{(i^*j^*)} q_{i^*j^*} \qquad \forall (i,j) \in A \setminus A_R, \quad \forall k \in K, \quad l = 1, \ldots, D_{max} \tag{20}$$

$$\sum_{d \in D} dT_d^l \geq \sum_{d \in D} dT_d^{l+1} \qquad \forall l = 1, \ldots, D_{max} - 1. \tag{21}$$

Constraints (19) and (20) impose lower bounds on the flow variables and constraints (21) break symmetric solutions regarding the link to a parking lot.

3.1.2. Park and loop model with flow formulation

In order to formulate flow constraints for the transfer vehicle, new variables s_{ij} are necessary to model the flow of the transfer vehicle. The flow formulation (PAL-flow) of the park and loop model then reads:

$$\text{(PAL-flow)} \ \min \sum_{l=1,\ldots,D_{max}} \left(\sum_{k \in K} \left(\sum_{(i,j) \in A_R} c_{ij}^{serv,k} x_{ij}^{k,l} + \sum_{(ij) \in A} c_{ij}^k y_{ij}^{k,l} \right) \right) + \sum_{i \in D'} \sum_{j \in D' \setminus \{i\}} SP_{ij} r_{ij}$$

$$+ \sum_{d' \in D'} \left(DC_{d'} + OC_{d'} z_{d'} \right) \tag{22}$$

$$\text{s.t. Constraints } (2) - (15) \text{ of PAL} \tag{23}$$

$$\sum_{j \in D' \setminus \{i\}} r_{ij} = z_i \quad \forall i \in D' \setminus \{d_L\}, \qquad \sum_{i \in D' \setminus \{j\}} r_{ij} = z_j \quad \forall j \in D' \setminus \{d_L\} \tag{24}$$

$$\sum_{j \in D} s_{ji} - \sum_{j \in D} s_{ij} = z_i \quad \forall i \in D \setminus \{d_L\} \tag{25}$$

$$\sum_{j \in D} s_{d_L j} = |D| - 1 \tag{26}$$

$$(|D| - 1) r_{ij} - s_{ij} \geq= 0 \quad \forall i,j \in D \text{ and } i \neq j \tag{27}$$

$$x^{k,l} \in \{0,1\}^{|A_R|}, \quad y^{k,l} \in \mathbb{Z}_+^{|A|}, \quad f_{ij}^{k,l} \geq 0 \quad \forall (i,j) \in A, \quad \forall k \in K, \quad l = 1, \ldots, D_{max}$$

$$z \in \{0,1\}^{|D'|}, \quad T^l \in \{0,1\}^{|D'|} \quad l = 1, \ldots, D_{max}$$

$$r_{ij} \in \{0,1\} \quad \forall i,j \in D', \quad s_{ij} \geq 0 \quad \forall (i,j) \in A. \tag{28}$$

The objective (22) and constraints (23) are the same as in the PAL model before. Constraints (24)–(27) model the tour of the transfer vehicle. Instead of adding generalized subtour elimination constraints (see (17)), a flow formulation is now used. Whenever node i is selected as an open parking lot, the transfer vehicle has to enter and leave that node, which is stated by constraints (24). Flow conservation is guaranteed by constraints (25) and (26). The first set of constraints (25) states that whenever node i is a parking lot, one unit of flow has to be absorbed by that node. The second constraint states that exactly $|D| - 1$ units of flow leave the loop-depot d_L. If less than $|D|$ parking lots are open (either $D_{max} < |D|$ or because of optimality), the remaining flow is taken by the loop variable $s_{d_L d_L}$. Constraints (27) link the flow variables s_{ij} with the transfer variables r_{ij}. The same improving constraints (19)–(21) as for the PAL model can be added.

3.2. Combined park and loop and curbline models

This paragraph will introduce two mixed integer formulations for the combined park and loop and curbline delivery mode. In contrast to the models of the previous paragraph, the postman is now also allowed to deliver mail with the transfer vehicle. Again, the two models differ in the way they handle infeasible subtours of the transfer vehicle. The first formulation follows the idea of the sparse formulation presented by Belenguer and Benavent (1998) for the CARP. Additional balance constraints resulting from the mixed network and constraints ensuring the connectivity of parking lots are added. The computational results of Belenguer and Benavent (1998) show that the formulation work well for small sized problems. The second formulation uses flow variables in the same spirit as presented in (Gouveia *et al.*, 2010). Their computation results show that this formulation is sometimes able to improve the lower bounds obtained by the aggregated version of the spare formulation.

3.2.1. Combined park and loop and curbline routes with generalized SEC

In order to formulate the model, new variables rx_e, rx_a, ry_e, ry_a, and p_i are introduced. Variables rx_e and rx_a take value 1 if the transfer vehicle services edge e or arc a. Variables ry_e and ry_a count the number of times an edge e or arc a is traversed without being serviced by the transfer vehicle. Auxiliary variables p_i for every node $i \in V$ are needed to ensure even node degrees. Note that the transfer vehicle is indexed by $k = 0$ and the set of (required) arcs of the original network is indicated by A' (A'_R). A mathematical formulation for the combined delivery mode is then:

(CurbPAL)

$$\min \sum_{l=1,\dots,D_{max}} \left(\sum_{k \in K} \left(\sum_{(i,j)\in A_R} c_{ij}^{serv,k} x_{ij}^{k,l} + \sum_{(ij)\in A} c_{ij}^{k} y_{ij}^{k,l} \right)\right) + \sum_{e \in E_R} c_e^{serv,0} rx_e + \sum_{a \in A'_R} c_a^{serv,0} rx_a + \sum_{e\in E} c_e^0 ry_e + \sum_{a \in A'} c_a^0 ry_a + \sum_{d \in D} (DC_d + OC_d z_d) \tag{29}$$

$$\text{s.t. Constraints (2) and (4)} - \text{(15) of PAL} \tag{30}$$

$$\sum_{l=1,\dots,D_{max}} \sum_{k\in K} x_{ij}^{k,l} + rx_a = 1 \quad \forall a = (i,j) \in A'_R,$$

$$\sum_{l=1,\dots,D_{max}} \sum_{k\in K} (x_{ij}^{k,l} + x_{ji}^{k,l}) + rx_e = 1 \quad \forall e = \{i,j\} \in E_R \tag{31}$$

$$\sum_{e \in \delta_R(d_L)} rx_e + \sum_{a \in \delta_R^+(d_L)} rx_a + \sum_{e\in\delta(d_L)} ry_e + \sum_{a \in \delta^+(d_L)} ry_a + z_{d_L} \geq 1 \tag{32}$$

$$\sum_{e\in E_R} rx_e + \sum_{a \in A_R} rx_a \leq Q^0 \tag{33}$$

$$\sum_{e\in\delta_R(i)} rx_e + \sum_{a\in\delta_R^+(i)\cup\delta_R^-(i)} rx_a + \sum_{e\in\delta(i)} ry_e + \sum_{a\in\delta^+(i)\cup\delta^-(i)} ry_a = 2p_i \quad \forall i \in V \tag{34}$$

$$-\sum_{a\in\delta_R^+(S)} rx_a - \sum_{a\in\delta^+(S)} ry_a + \sum_{a\in\delta_R^-(S)} rx_a + \sum_{a\in\delta^-(S)} ry_a + \sum_{e\in\delta_R(s)} rx + \sum_{e\in\delta(S)} ry_a \geq 0 \quad \forall S \subseteq V \tag{35}$$

$$\sum_{e\in\delta_R(i)} rx_e + \sum_{a\in\delta_R^+(i)\cup\delta_R^-(i)} rx_a + \sum_{e\in\delta(i)} ry_e + \sum_{a\in\delta^+(i)\cup\delta^-(i)} ry_a \geq \begin{cases} 2rx_e, & e \in E_R(S) \\ 2rx_a, & a \in A'_R(S) \\ 2ry_e, & e \in E(S) \\ 2ry_a, & a \in A'(S) \\ 2z_i, & i \in D' \cap S \end{cases} \quad \forall S \subseteq V \setminus \{d_L\} \tag{36}$$

$$x^{k,l} \in \{0,1\}^{|A_R|}, \quad y^{k,l} \in \mathbb{Z}_+^{|A|}, \quad rx \in \{0,1\}^{|A'_R \cup E_R|}, \quad ry \in \mathbb{Z}_+^{|A' \cup E|}, p \in \mathbb{Z}_+^{|V|}$$
$$f^{k,l} \geq 0, \quad \forall k \in K,\ l = 1,\dots,D_{max}, \quad z \in \{0,1\}^{|D|}, \quad T^l \in \{0,1\}^{|D|} \quad l = 1,\dots,D_{max}. \tag{37}$$

The objective (29) calculates the costs that occur when either a vehicle of the smaller delivery routes starting at a parking lot or the transfer vehicle services or traverses an edge or arc. Again, hiring costs for vehicles in use and fixed costs for opening a parking lot are taken into account. Constraints (30) are the same as for the pure park and loop delivery mode and model feasible routes for vehicles of the walking loops. The service constraints (31) take into account that it is now possible to service an edge or arc with the transfer vehicle. A feasible route for the transfer vehicle is defined by constraints (32)–(36). In detail:

- Constraint (32) ensures that if the loop-depot d_L is not chosen as an open parking lot, at least one outgoing edge or arc of the transfer vehicle has to be used.
- Constraint (33) ensures the capacity restriction of the transfer vehicle.
- Constraints (34) ensure that for every node $i \in V$, the sum of incoming and outgoing edges and arcs is even.
- Constraints (35) are balance constraints and ensure that for every subset of nodes the difference between incoming and outgoing arcs can be compensated by edges.
- Subtour elimination constraints (36) ensure that there are no disconnected arcs or edges and no disconnected open parking lots from the loop-depot d_L.

As before, improving constraints for bounding the flow of vehicles in the walking loop (19), (20) and symmetry breaking constraints (21) can be added to speed up the solution process.

3.2.2. Combined park and loop and curbline routes with flow formulation

The second formulation of the combined park and loop and curbline routes uses again a flow formulation to describe a feasible route of the transfer vehicle. For each arc $(i, j) \in A'$, there is a variable ry_{ij} counting the number of traversings of the transfer vehicle without servicing. Whenever the arc (i, j) is required, a binary variable rx_{ij} exists taking value 1 if (i, j) is serviced by the transfer vehicle, and 0 otherwise. Two service and deadheading variables rx_{ij}, rx_{ji} and ry_{ij}, ry_{ji}, respectively, exist for each edge $\{i, j\}$ representing both directions in which an edge can be traversed. Similar, flow variables s_{ij} exist, one for each arc (i, j) and two for each edge $\{i, j\}$. The flow formulation of the combined PAL with curbline problem then reads:

(CurbPAL-flow)

$$\begin{aligned} \min \sum_{l=1,\ldots,D_{max}} & \left(\sum_{k\in K} \left(\sum_{(i,j)\in A_R} c_{ij}^{serv,k} x_{ij}^{k,l} + \sum_{(ij)\in A} c_{ij}^{k} y_{ij}^{k,l} \right) \right) \\ & + \sum_{(i,j)\in A_R} c_{ij}^{serv,0} rx_{ij} + \sum_{(i,j)\in A} c_{ij}^{0} ry_{ij} \\ & + \sum_{d\in D} (DC_d + OC_d z_d) \end{aligned} \tag{38}$$

$$\text{s.t. Constraints (2) and (4) -- (15) of PAL} \tag{39}$$

$$\sum_{l=1,\ldots,D_{max}} \sum_{k\in K} x_{ij}^{k,l} + rx_{ij} = 1 \quad \forall (i,j) \in A'_R,$$

$$\sum_{l=1,\ldots,D_{max}} \sum_{k\in K} (x_{ij}^{k,l} + x_{ji}^{k,l}) + (rx_{ij} + rx_{ji}) = 1 \quad \forall \{i,j\} \in E_R \tag{40}$$

$$\sum_{j\in\delta^+(i)} ry_{ij} + \sum_{\delta_R^+(i)} rx_{ij} = \sum_{\delta^-(i)} ry_{ji} + \sum_{\delta_R^-(i)} rx_{ji} \quad \forall i \in V \tag{41}$$

$$\sum_{j\in\delta^-(i)} s_{ji} - \sum_{j\in\delta^+(i)} s_{ij} = \sum_{j\in\delta_R^-(i)} q_{ji} rx_{ji} \quad \forall i \in V \tag{42}$$

$$\sum_{j\in\delta^+(d_L)} s_{d_L j} = \sum_{(i,j)\in A_R} q_{ij} rx_{ij} \tag{43}$$

$$s_{ij} \leq Q^0(rx_{ij} + ry_{ij}) \qquad \forall (i,j) \in A \qquad (44)$$

$$\sum_{j \in \delta^+(d')} rx_{d'j} + \sum_{j \in \delta^+(d')} ry_{d'j} \geq z_{d'} \qquad \forall d' \in D' \qquad (45)$$

$$x^{k,l} \in \{0,1\}^{|A_R|}, \quad y^{k,l} \in \mathbb{Z}_+^{|A|}, \qquad rx \in \{0,1\}^{|A'_R \cup E_R|}, \quad ry \in \mathbb{Z}_+^{|A' \cup E|}, \quad s \geq 0$$

$$f^{k,l} \geq 0, \quad \forall k \in K,\ l = 1, \dots, D_{max}, \quad z \in \{0,1\}^{|D|}, \quad T^l \in \{0,1\}^{|D|} \quad l = 1, \dots, D_{max}. \qquad (46)$$

The objective (38) and constraints (39) are the same as in the PAL model of the previous section. The service constraints (40) take into account that it is possible to service an edge or arc with the transfer vehicle. Because of the flow formulation, there are two variables associated with one edge: one for each possible traversing direction. A feasible route for the transfer vehicle is defined by constraints (41)–(45), which are formulated with flow variables. In detail:

- Constraints (41) ensure the continuity of the transfer vehicle.
- Constraints (42) ensure that the difference between incoming and outgoing flow of the transfer vehicle at node i is exactly the demand absorbed by the demand of arcs entering node i and serviced with the transfer vehicle. These generalized flow conservation constraints are similar to constraints (5) of the PAL model.
- Constraint (43) ensures that the outgoing flow of the loop-depot d_L equals the demand delivery by the transfer vehicle.
- Constraints (44) are upper bounds on the flow variables.
- Constraints (45) ensure that the transfer vehicle reaches every open parking lot.

As before, improving constraints for bounding the flow of vehicles in the walking loop (19), (20) and symmetry breaking constraints (21) can be added to speed up the solution process.

4. Computational results

This section reports computational results for the four models presented for the park and loop problem and the combination with curbline delivery. To test the quality of the new formulations, we randomly generated a benchmark set of 39 instances on mixed graphs with both required and non-required edges and arcs. For instances `Pal1` to `Pal25`, the size of the underlying network increases for both nodes and edges or arcs. There are three types of vehicles available, where the first type always represents the transfer vehicle. It is assumed that there exists just one transfer vehicle. Instances `Pal26` to `Pal39` model problems with two or three districts that are connected by arcs or edges. These instances are derived by combining some of the first problems and additional arcs and edges to ensure a strongly connected graph. Again, three different vehicle types are given, but more vehicles per type are available. Service and transfer costs are provided for every link and every vehicle type. About 20% of the nodes represent possible parking lots.

All computations were performed on a standard PC with an Intel©Core^{TM} i7-2600 processor at 3.4 GHz with 16 GB of main memory. The four models for the (combined) park and loop problem were introduced to CPLEX through the callable C++ API of CPLEX 12.2 and the cutting plane algorithm was coded in C++ (MS-Visual Studio, 2010). For separating violated SEC in the (Curb)PAL model, we follow the idea of Benavent *et al.* (2000) and compute a min-cut separating the loop-depot d_L and a parking lot h or an edge or arc traversed or serviced by the transfer vehicle. Separating violated balance set constraints is a bit more tricky to implement. The separation procedure we follow is described in (Nobert and Picard, 1996). A hard time limit of four hours has been set for CPLEX to solve the model. We also use CPLEX for finding an upper bound and applied the feasibility pump heuristic with an emphasis on finding a feasible solution.

Tables 1–4 report results of the linear relaxation and the end of the branch-and-bound tree on all tested benchmark sets. The entries of the header of all tables have the following meaning:

$D_{max} = 2,3$	maximum number of open parking lots
(Curbline+) PAL/PAL-flow	lower bounds and computation times for the PAL model (1)–(17), the PAL-flow model (22)–(27) the combined park and loop with curbline mode CurbPAL model (29)–(36) and the CurbPAL-flow model (38)–(44)
instance	name of the instance
$\|V\|$,$\|A \cup E\|$,$\|A_R\|$,$\|E_R\|$,P_{total}	characteristics of the instance: $\|V\|$ number of nodes, $\|A \cup E\|$ number of links, $\|A_R\|$ number of required arcs, $\|E_R\|$ number of required edges, P_{total} total number of vehicles at each parking lot Due to the sake of brevity, this information is omitted in the reporting integer results.
lb	lower bound at the end of the root node or the end of the branch-and-bound tree
time	computation time in seconds; if the time limit is reached, it is indicated by *4h*
lb_{best}	best lower bound obtained with either the PAL or PAL-flow formulation or CurbPAL or CurbPAL-flow formulation for the combined problem
ub	best upper bound reported by CPLEX

The following additional information is given for the respective model:

Num lb_{best}	number of instances that provided the best lower bound lb_{best}
Num opt	number of obtained integer solutions
avg time	average computation time for the root node or the whole branch-and-bound tree

4.1. Linear Relaxation Results

We will start with the analysis of the linear relaxation results for both delivery modes obtained at the end of the root node. Tables 1 and 2 show the results for the generated `Pal` instances.

Comparing the values of the lower bounds for the pure park and loop delivery mode, the performance of a formulation seems to depend on the problem size. If at maximum two parking lots can be opened, the number of best lower bounds lb_{best} is slightly higher for the PAL-flow model than for the PAL model. However, allowing at maximum three open parking lots, the problem size increases by both $\mathcal{O}(|A|P_{total})$ in variables and constraints. Then, the PAL model formulated with generalized SEC results more often in better lower bounds than does the flow formulation PAL-flow. Overall, the computation time for solving the root node is on average higher for the PAL model than for the PAL-flow model.

Comparing the linear relaxation results for the combined park and loop with curbline mode in Table 2, we see that the PAL-flow model clearly outperforms the PAL model. For both $D_{max} = 2$ and $D_{max} = 3$, higher lower bounds are obtained with the PAL-flow formulation in 31 and 32 out of 39 cases, respectively. Also, the average computation time is by factor 18 and 25, respectively, drastically smaller than the average computation time of the root node with the PAL model.

The combined park and loop with curbline mode is a more complex problem than the pure park and loop mode, as additional decisions on whether or not to service an edge or arc with the transfer vehicle are required. Increasing computation times for the root node of the combined problem modeled with the CurbPAL formulation support this. Surprisingly, solving the root node of the CurbPAL-flow formulation is much faster than the PAL-flow formulation of the pure park and loop mode.

4.2. Integer Results

In Tables 3 and 4, we report results at the end of the branch-and-bound tree. Again, the flow formulation

Table 1: Lower bounds for `Pal` instances with park and loop mode

						$D_{max}=2$					$D_{max}=3$				
						PAL		PAL-flow			PAL		PAL-flow		
instance	$\|N\|$	$\|A \cup E\|$	$\|A_R\|$	$\|E_R\|$	P_{total}	lb	time	lb	time	lb_{best}	lb	time	lb	time	lb_{best}
Pal1	9	14	6	6	6	403	0.0	403	0.1	403	403	0.1	403	0.2	403
Pal2	9	17	9	5	6	470	0.3	453	0.2	470	470	0.4	468	0.3	470
Pal3	9	13	5	7	6	317	1.9	316	0.2	317	328	3.8	326	0.2	328
Pal4	10	15	7	9	6	279	0.1	288	0.2	288	279	0.6	279	0.2	279
Pal5	9	17	8	8	8	431	1.3	434	0.4	434	431	3.9	423	0.6	431
Pal6	36	53	38	33	6	1711	7.0	1720	7.1	1720	1714	15.9	1708	14.1	1714
Pal7	36	46	32	32	6	1913	8.5	1904	5.6	1913	1903	13.8	1905	8.4	1905
Pal8	36	46	32	34	6	1763	16.5	1764	9.0	1764	1754	72.2	1747	47.1	1754
Pal9	36	48	34	32	6	1573	7.6	1568	7.0	1573	1555	21.2	1561	17.4	1561
Pal10	36	60	45	30	6	1839	7.9	1858	7.1	1858	1836	13.2	1835	7.5	1836
Pal11	37	66	20	18	6	1236	6.5	1232	4.1	1236	1226	16.7	1214	7.4	1226
Pal12	36	66	21	18	6	1138	2.7	1151	2.6	1151	1136	9.0	1137	6.8	1137
Pal13	36	63	19	17	6	1146	7.0	1148	4.4	1148	1138	14.4	1138	9.3	1138
Pal14	37	69	21	21	6	1082	6.8	1086	5.2	1086	1088	21.3	1074	9.4	1088
Pal15	36	68	21	18	8	1311	8.3	1316	4.6	1316	1301	12.9	1301	7.4	1301
Pal16	81	111	85	81	6	4314	238.2	4319	85.2	4319	4268	371.4	4254	110.6	4268
Pal17	81	102	77	82	6	4320	184.3	4317	105.6	4320	4284	319.5	4260	77.8	4284
Pal18	81	105	81	77	6	4655	112.4	4614	39.5	4655	4622	595.9	4621	179.6	4622
Pal19	81	112	87	76	6	3963	108.4	3939	50.5	3963	3930	126.9	3924	85.7	3930
Pal20	81	105	81	77	6	4156	345.9	4124	61.0	4156	4114	390.3	4107	55.2	4114
Pal21	81	140	44	44	6	4958	54.5	4949	21.9	4958	4946	315.3	4946	87.1	4946
Pal22	81	143	46	43	6	2823	105.7	2822	68.4	2823	2817	474.8	2816	237.5	2817
Pal23	81	140	43	46	6	2783	133.9	2787	77.2	2787	2757	470.3	2759	304.8	2759
Pal24	81	142	46	44	6	3195	185.0	3203	56.8	3203	3169	170.6	3181	187.8	3181
Pal25	81	136	40	48	6	2489	55.7	2515	72.0	2515	2474	347.1	2472	171.6	2474
Pal26	18	27	15	11	7	830	4.1	834	2.9	834	830	4.9	829	3.7	830
Pal27	18	24	12	13	6	777	3.9	777	0.6	777	766	4.5	768	1.5	768
Pal28	18	26	14	13	5	803	4.1	812	2.5	812	796	4.8	780	2.7	796
Pal29	18	30	16	14	6	679	4.6	679	2.9	679	667	4.3	668	3.1	668
Pal30	18	28	16	15	8	763	4.8	766	2.7	766	760	5.9	762	4.4	762
Pal31	27	47	25	19	12	1187	12.0	1185	7.1	1187	1185	25.9	1185	13.5	1185
Pal32	27	40	23	19	11	1251	9.0	1255	6.1	1255	1238	26.8	1237	8.1	1238
Pal33	54	74	50	49	13	2271	43.2	2272	196.4	2272	2262	81.3	2262	283.2	2262
Pal34	27	43	23	23	11	951	8.4	946	3.6	951	947	10.0	946	6.1	947
Pal35	54	78	53	50	13	2500	101.6	2472	53.7	2500	2462	143.4	2466	112.0	2466
Pal36	54	72	48	48	16	2377	752.4	2381	934.7	2381	2365	1458.4	2360	1314.7	2365
Pal37	54	71	45	48	16	2364	167.4	2353	101.4	2364	2350	686.3	2376	1465.2	2376
Pal38	27	44	24	23	8	1159	14.2	1161	10.2	1161	1157	32.4	1158	13.7	1158
Pal39	54	88	61	45	6	2538	13.3	2534	6.3	2538	2534	59.1	2527	9.0	2534
Num lb_{best}						19		23			28		18		
avg time							70.5		52.0			162.8		125.0	

Table 2: Lower bounds for `Pal` instances with combined park and loop with curbline mode

						$Dmax = 2$					$Dmax = 3$				
						CurbPAL		CurbPAL-flow			CurbPAL		CurbPAL-flow		
instance	$\lvert N\rvert$	$\lvert A \cup E\rvert$	$\lvert A_R\rvert$	$\lvert E_R\rvert$	P_{total}	lb	time	lb	time	lb_{best}	lb	time	lb	time	lb_{best}
Pal1	9	14	6	6	6	395	0.5	395	0.2	395	395	1.6	395	0.1	395
Pal2	9	17	9	5	6	433	0.7	454	0.3	454	427	0.8	453	0.3	453
Pal3	9	13	5	7	6	307	0.4	317	0.2	317	312	2.0	317	0.2	317
Pal4	10	15	7	9	6	286	1.6	280	0.8	286	274	0.7	273	0.5	274
Pal5	9	17	8	8	8	408	0.4	418	0.4	418	405	4.6	408	0.5	408
Pal6	36	53	38	33	6	1663	68.7	1665	9.4	1665	1659	193.4	1671	9.0	1671
Pal7	36	46	32	32	6	1856	44.2	1867	5.9	1867	1855	114.8	1850	9.2	1855
Pal8	36	46	32	34	6	1689	56.9	1693	7.0	1693	1673	145.5	1687	8.8	1687
Pal9	36	48	34	32	6	1529	27.2	1550	7.4	1550	1530	86.3	1544	6.4	1544
Pal10	36	60	45	30	6	1792	44.8	1801	6.4	1801	1786	116.3	1815	10.8	1815
Pal11	37	66	20	18	6	1192	27.2	1191	5.3	1192	1170	34.5	1178	6.1	1178
Pal12	36	66	21	18	6	1111	14.9	1131	5.9	1131	1107	31.4	1113	7.1	1113
Pal13	36	63	19	17	6	1126	30.1	1135	7.8	1135	1123	53.7	1129	14.3	1129
Pal14	37	69	21	21	6	1046	36.1	1039	5.7	1046	1042	70.5	1036	6.2	1042
Pal15	36	68	21	18	8	1275	24.6	1274	4.7	1275	1275	40.7	1284	8.8	1284
Pal16	81	111	85	81	6	4124	3116.0	4135	35.4	4135	4101	6632.5	0	0.0	4101
Pal17	81	102	77	82	6	4180	2241.4	4217	181.5	4217	4154	6922.4	4198	82.9	4198
Pal18	81	105	81	77	6	4530	2241.0	4541	154.8	4541	4511	4540.4	4535	423.4	4535
Pal19	81	112	87	76	6	3849	2419.7	3864	47.1	3864	3829	4922.7	3837	91.4	3837
Pal20	81	105	81	77	6	4025	2421.3	4061	152.1	4061	4008	4660.3	4027	180.3	4027
Pal21	81	140	44	44	6	4862	872.8	4888	13.5	4888	4858	2253.5	4858	38.7	4858
Pal22	81	143	46	43	6	2693	1055.1	2724	51.4	2724	2687	2902.2	2700	94.4	2700
Pal23	81	140	43	46	6	2677	1758.7	2698	246.6	2698	2647	3469.8	2655	321.6	2655
Pal24	81	142	46	44	6	3077	909.3	3098	0.0	3077	3069	2036.3	3072	134.5	3072
Pal25	81	136	40	48	6	2426	1071.1	2439	66.6	2439	2402	2667.3	2426	231.0	2426
Pal26	18	27	15	11	7	555	4.5	555	2.5	555	555	5.3	555	2.6	555
Pal27	18	24	12	13	6	569	4.4	569	1.0	569	568	5.4	569	1.6	569
Pal28	18	26	14	13	5	558	5.2	552	2.6	558	558	7.2	555	2.7	558
Pal29	18	30	16	14	6	548	5.0	550	2.5	550	545	6.6	545	3.0	545
Pal30	18	28	16	15	8	616	6.6	620	3.2	620	616	9.5	620	3.7	620
Pal31	27	47	25	19	12	888	12.2	890	4.0	890	889	29.5	890	5.7	890
Pal32	27	40	23	19	11	906	19.2	915	1.9	915	898	28.4	901	5.3	901
Pal33	54	74	50	49	13	1895	941.1	1892	17.1	1895	1880	1864.7	1888	28.8	1888
Pal34	27	43	23	23	11	811	12.9	816	3.9	816	811	23.2	810	5.5	811
Pal35	54	78	53	50	13	2026	978.0	2031	34.5	2031	2026	2366.4	2026	34.0	2026
Pal36	54	72	48	48	16	1929	998.4	1940	94.6	1940	1932	2672.5	1935	81.0	1935
Pal37	54	71	45	48	16	1850	1430.6	1858	19.4	1858	1851	1802.8	1851	101.9	1851
Pal38	27	44	24	23	8	938	19.3	937	4.1	938	937	36.3	937	5.0	937
Pal39	54	88	61	45	6	2109	169.6	2112	8.9	2112	2109	435.3	2108	11.4	2109
Num lb_{best}						10		32			13		33		
avg time							592.1		33.0			1312.7		51.9	

Table 3: Integer results for `Pal` instances with park and loop mode

	$D_{max}=2$							$D_{max}=3$						
	PAL			PAL-flow				PAL			PAL-flow			
instance	*lb*	*ub*	time	*lb*	*ub*	time	lb_{best}	*lb*	*ub*	time	*lb*	*ub*	time	lb_{best}
Pal1	opt		0.1	opt		0.1	403	opt		0.3	opt		0.1	403
Pal2	opt		0.5	opt		0.2	472	opt		0.8	opt		0.7	472
Pal3	opt		2.7	opt		0.1	328	opt		3.7	opt		0.1	328
Pal4	opt		3.7	opt		0.5	303	opt		24.8	opt		0.7	303
Pal5	opt		6.9	opt		1.3	444	opt		14.1	opt		1.3	444
Pal6	1734	1740	*4h*	opt		35.8	1740	1725	1740	*4h*	opt		149.4	1740
Pal7	opt		1753.3	opt		28.6	1944	opt		8109.3	opt		75.4	1944
Pal8	opt		102.6	opt		18.3	1770	opt		924.0	opt		80.5	1770
Pal9	opt		99.4	opt		14.6	1586	opt		6791.4	opt		31.0	1586
Pal10	1867	1883	*4h*	opt		46.3	1883	1849	1883	*4h*	opt		316.4	1883
Pal11	opt		880.7	opt		22.8	1258	opt		9273.1	opt		57.8	1258
Pal12	opt		1908.8	opt		35.1	1186	opt		11346.5	opt		103.9	1186
Pal13	opt		237.6	opt		43.9	1186	opt		7415.7	opt		292.8	1186
Pal14	opt		10341.2	opt		90.7	1132	1105	1136	*4h*	opt		368.5	1132
Pal15	opt		131.6	opt		21.0	1328	opt		620.9	opt		71.9	1328
Pal16	4328	4345	*4h*	opt		5311.6	4344	4268	4668	*4h*	4316	4511	*4h*	4316
Pal17	4334	4420	*4h*	opt		4771.6	4361	4292	4549	*4h*	4343	4371	*4h*	4343
Pal18	4660	4806	*4h*	4700	4726	*4h*	4700	4623	5224	*4h*	4668	4879	*4h*	4668
Pal19	3972	4004	*4h*	opt		6944.7	3999	3934	4269	*4h*	3987	3999	*4h*	3987
Pal20	4162	4333	*4h*	4179	4297	*4h*	4179	4120	4426	*4h*	4163	4318	*4h*	4163
Pal21	5005	5006	*4h*	opt		302.7	5006	4958	5017	*4h*	opt		1838.6	5006
Pal22	2828	3074	*4h*	opt		7515.2	2870	2820	2892	*4h*	2838	2924	*4h*	2838
Pal23	2802	2833	*4h*	opt		4879.8	2821	2769	2822	*4h*	opt		11538.7	2821
Pal24	3199	3291	*4h*	3262	3279	*4h*	3262	3171	3687	*4h*	3215	3291	*4h*	3215
Pal25	2503	2617	*4h*	opt		379.3	2559	2489	2683	*4h*	opt		8757.8	2559
Pal26	opt		351.0	opt		16.5	899	opt		3769.8	opt		24.2	899
Pal27	opt		8.8	opt		2.1	805	opt		179.2	opt		4.8	805
Pal28	opt		283.9	opt		12.5	848	841	848	*4h*	opt		114.6	848
Pal29	opt		10.1	opt		6.3	682	opt		51.6	opt		2.8	682
Pal30	opt		860.5	opt		33.6	776	775	776	*4h*	opt		40.5	776
Pal31	1203	1307	*4h*	1253	1266	*4h*	1253	1188	1295	*4h*	1224	1347	*4h*	1224
Pal32	1274	1299	*4h*	opt		2305.8	1299	1243	1300	*4h*	opt		6951.5	1299
Pal33	2271		*4h*	2293	2377	*4h*	2293	2262	2659	*4h*	2280		*4h*	2280
Pal34	984	1009	*4h*	opt		1762.5	1009	972	1021	*4h*	opt		3060.7	1009
Pal35	2500		*4h*	2483	2684	*4h*	2500	2468		*4h*	2472	2639	*4h*	2472
Pal36	2378		*4h*	2394		*4h*	2394	2370		*4h*	2366		*4h*	2370
Pal37	2365		*4h*	2369	3182	*4h*	2369	2351	3356	*4h*	2362		*4h*	2362
Pal38	1175	1184	*4h*	opt		155.0	1184	1174	1184	*4h*	opt		1326.5	1184
Pal39	2549	2566	*4h*	opt		6964.7	2564	2541	2570	*4h*	2557	2564	*4h*	2557
Num lb_{best}	19			38				16			38			
Num opt	18			31				15			26			
avg time			8189.3			4023.7				10105.8			5702.9	

Table 4: Integer results for `Pal` instances with combined park and loop with curbline mode

	$D_{max}=2$							$D_{max}=3$						
	CurbPAL			CurbPAL-Flow				CurbPAL			CurbPAL-Flow			
instance	*lb*	*ub*	time	*lb*	*ub*	time	lb_{best}	*lb*	*ub*	time	*lb*	*ub*	time	lb_{best}
Pal1	opt		0,4	opt		0,5	395	opt		1,2	opt		0,1	395
Pal2	opt		10.9	opt		8.1	454	opt		14.6	opt		0.6	454
Pal3	opt		1.8	opt		0.3	317	opt		2.0	opt		0.1	317
Pal4	opt		10.3	opt		11.6	298	opt		88.8	opt		6.8	298
Pal5	opt		126.7	opt		4.8	431	opt		167.9	opt		189.7	431
Pal6	1663	1735	*4h*	1689	1690	*4h*	1689	1654	1910	*4h*	1676	1690	*4h*	1676
Pal7	opt		796.2	opt		40.2	1874	1854	1878	*4h*	opt		136.8	1874
Pal8	1707	1735	*4h*	opt		7057.4	1727	1677	2033	*4h*	opt		2648.1	1727
Pal9	1539	1563	*4h*	opt		113.3	1561	1528	1561	*4h*	opt		3169.9	1561
Pal10	1798	1874	*4h*	1824	1830	*4h*	1824	1785		*4h*	opt		1528.2	1830
Pal11	1204	1213	*4h*	opt		890.0	1213	1191	1216	*4h*	opt		3473.5	1213
Pal12	1138	1149	*4h*	opt		87.7	1149	1126	1152	*4h*	opt		311.2	1149
Pal13	opt		10042.7	opt		6507.3	1179	1147	1179	*4h*	1167	1179	*4h*	1167
Pal14	1063	1114	*4h*	1075	1112	*4h*	1075	1047	1141	*4h*	1064	1109	*4h*	1064
Pal15	1285	1302	*4h*	opt		92.0	1302	1278	1304	*4h*	opt		33.0	1302
Pal16	4122		*4h*	4166	4332	*4h*	4166	4106		*4h*	4144		*4h*	4144
Pal17	4172		*4h*	4235	4445	*4h*	4235	4157		*4h*	4207		*4h*	4207
Pal18	4532		*4h*	4577	4833	*4h*	4577	4514		*4h*	4553	4764	*4h*	4553
Pal19	3836		*4h*	3887	3950	*4h*	3887	3835		*4h*	3856	4138	*4h*	3856
Pal20	4036		*4h*	4073	4243	*4h*	4073	4010		*4h*	4040		*4h*	4040
Pal21	4881	4903	*4h*	opt		220.0	4902	4869	4936	*4h*	4894	4902	*4h*	4894
Pal22	2694		*4h*	2730	2793	*4h*	2730	2688		*4h*	2718		*4h*	2718
Pal23	2682		*4h*	2715	2924	*4h*	2715	2659		*4h*	2687	3008	*4h*	2687
Pal24	3088	3796	*4h*	3120	3229	*4h*	3120	3069		*4h*	3098	3420	*4h*	3098
Pal25	2436		*4h*	2464	2593	*4h*	2464	2399		*4h*	2448	2616	*4h*	2448
Pal26	opt		4.9	opt		2.9	555	opt		4.9	opt		2.8	555
Pal27	opt		5.8	opt		2.1	569	opt		10.8	opt		1.7	569
Pal28	opt		15.7	opt		9.4	561	opt		39.6	opt		7.1	560
Pal29	opt		4.2	opt		2.9	550	opt		70.3	opt		7.3	550
Pal30	opt		29.3	opt		4.2	621	opt		106.8	opt		5.0	621
Pal31	opt		96.8	opt		12.9	900	893	900	*4h*	opt		30.6	900
Pal32	opt		205.5	opt		3.8	915	911	915	*4h*	opt		30.8	915
Pal33	1884		*4h*	1901	1942	*4h*	1901	1879		*4h*	1894	2107	*4h*	1894
Pal34	825	842	*4h*	opt		4977.1	839	814	838	*4h*	826	832	*4h*	826
Pal35	2026	2292	*4h*	2039	2164	*4h*	2039	2026		*4h*	2033	2070	*4h*	2033
Pal36	1939		*4h*	1956	1971	*4h*	1956	1931		*4h*	1947		*4h*	1947
Pal37	1851		*4h*	1871	1930	*4h*	1871	1845	2102	*4h*	1862		*4h*	1862
Pal38	opt		620.7	opt		95.2	945	939	945	*4h*	opt		113.6	945
Pal39	2111		*4h*	opt		5799.1	2125	2104		*4h*	2119	2141	*4h*	2119
Num lb_{best}	15			39				10			39			
Num opt	15			23				10			20			
avg time			9168.5			6572.9				10720.7			7315.3	

of the PAL problem clearly outperforms the formulation with the generalized SEC. As reported in Table 3, the PAL-flow model results in 38 out of 39 cases in a better lower bound. The only exceptions are instances `Pal35` for $D_{max} = 2$ and `Pal36` for $D_{max} = 3$. Also, the number of obtained optimal integer solutions is much higher for the flow formulation (31 and 26, respectively). The advantage of the PAL-Flow model is also shown by the smaller average computation times (about factor two times faster). Similar results are obtained for the combined park and loop and curbline mode. There, all 39 instances for both $D_{max} = 2$ and $D_{max} = 3$ are solved best with the flow formulation. All integer solutions found with the CurbPAL-flow model are also found with the CurbPAL model, but not vice versa.

5. Conclusion

In this paper, we have considered the different mail delivery modes classified by Bodin and Levy (2000) and have assigned the existing literature on LARPs to these categories. We have also presented mathematical formulations for two of the delivery modes, and showed computational results for all formulations. Extending the standard MCARP by a location aspect results in a combined location and arc routing problem. However, there are several possibilities for how locations are connected among themselves and with delivery routes.

First, the existing literature concerned with location arc routing is reviewed to distinguish between these possibilities. Considering the mail delivery context, these extensions can be described by different delivery modes.

Second, when the postman drives a transfer vehicle that allows a service to be performed or not, the corresponding delivery mode is called park and loop mode or combined park and loop with curbline mode, respectively. For each of these two delivery modes, we have proposed two mathematical formulations. The two formulations differ in how they model feasible routes of the transfer vehicle. In particular, the first model uses generalized subtour elimination constraints, where missing constraints are identified by a cutting plane procedure and added dynamically. This has the advantage of starting with a small problem and adding only relevant constraints. The second model uses a flow formulation to model feasible routes of the transfer vehicle. The advantage is that only a linear number of constraints is needed to describe feasible routes. Therefore, the whole model can be stated at once. As far as we know, this is the first time that a mathematical formulation both for the park and loop and the combined park and loop with curbline delivery has been presented.

Third, we have provided computational results, where the performance of the different formulations is analyzed. The results show that the flow formulation of both problem types clearly outperforms the formulation with generalized subtour elimination constraints. Both better bounds and lower computation times are obtained with the flow formulation.

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