A Note on Postman Problems with Zigzag Service

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Abstract

This note presents a generalization of postman problems with more flexibility of servicing street segments. Street segments requiring a service on both sides of the street can be covered either by two separate services or by a single zigzag service. A mixed integer formulation and a transformation of these postman problems into a symmetric traveling salesman problem are presented. Even if the cost for zigzag service is not smaller than the cost of two separate services, there is still a potential of improving solutions of the corresponding arc routing problem.

Key words: postman problems, arc routing, transformation

1 Introduction

This note presents a new class of postman problems which can be defined as follows. A postman has to deliver mail to the street segments of his district. Street segments can be divided into four classes: First, street segments with houses on one side of the street only. These require a single service. Second, street segments with houses on both sides, which have to be serviced separately. Third, street segments with houses on both sides which provide the option to service both sides with a single zigzag walk (going through the street segment once) or to service the two sides separately. In all cases, additional traversals of a segment are allowed, but a traversal and different modes of service imply different costs. Fourth, so-called non-required street segments may be used to get from one point to another. The problem is to find a least cost postman tour which provides appropriate service for all street segments of the

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given district. Literatur on arc routing, in which zigzag service has been mentioned, is scarce (Bodin *et al.* 1989, p. 49–51; Assad and Golden 1995, p. 436; Sniezek *et al.* 2002; zigzag service is called "meander" there). However, it seems that in this paper street segments are simply declared as "zigzag streets" so that zigzag service *options* are not considered in a model or corresponding algorithm.

One of the most general postman problems studied in the literature is the windy rural postman problem (WRPP, see Corberán *et al.* 2000), which covers the undirected, directed and mixed cases as well as the Chinese and rural cases. The windy case is important for several real-world applications that consider partially directed street networks with either symmetric or asymmetric costs. For a general introduction to postman problems and arc routing we refer to Eiselt *et al.* (1995b,a) and Dror (2000).

We define the *windy rural postman problem with zigzag service* (WRPPZ) as a generalization of the WRPP. An instance of the WRPPZ is defined on an undirected graph $G = (V, E)$ with node set V and edge set E. Without loss of generality one can assume that G is simple, i.e., G does not contain parallel edges or loops. Edges are partitioned into $E = E^0 \cup E^1 \cup E^2 \cup E^3$ where E^0 are the non-required edges, E^1 and E^2 the edges requiring single resp. double service (but zigzag service is not allowed), and E^3 edges that provide the zigzag service option. Up to eight different costs c^k_{ij}, c^k_{ji} with $k \in \{0, 1, 2, 3\}$ are associated with an edge $\{i, j\} \in E$. Herein, $k = 0$ stands for traversal, $k = 1$ for servicing the first side, $k = 2$ the opposite side of the street segment, and $k = 3$ for zigzag service, while ij and ji describe the orientation of the service/traversal. Obviously, for the edges $e \in E \setminus E^3$ only some of the costs are relevant, i.e., edges $e \in E^2$ have six, edges $e \in E^1$ have four, and edges $e \in E^0$ have two different costs.

Example 1 Consider the symmetric cost case and a graph with $V = \{i, j, k\}$ and $E = E^1 \cup E^3$ with $E^1 = \{\{i, j\}\}$ and $E^3 = \{\{j, k\}\}$. Independent of cost for different modes of service, the cost minimal postman tour is (i, j, k, j, i). Its cost is $c^1_{ij} + c^0_{ij} + \min\{c^0_{jk} + c^3_{jk}, c^1_{jk} + c^2_{jk}\}$. Normally, one would expect that traversal and zigzag service together are more costly than two separate services. Hence, this is an example where zigzagging is not advantageous.

Example 2 For a graph with nodes $V = \{i, j, k\}$ and edges $E = E^1 \cup E^3$ with $E^1 = \{\{i, j\}, \{i, k\}\}$ and $E^3 = \{\{j, k\}\}$ and symmetric costs, different postman tours may be optimal depending on the choice of the costs. First, the tour $T_1 = (i, j, k, i)$ services edge $\{j, k\} \in E^3$ with zigzag service and its cost is $c^1_{ij} + c^3_{jk} + c^1_{ik}$. Second, the tour $T_2 = (i, j, k, j, k, j, i)$ covers edge $\{j, k\} \in E^3$ by two separate services and performs an additional traversal trough the edge $\{j, k\}$ so that its cost is $c^1_{ij} + c^0_{jk} + c^1_{jk} + c^2_{jk} + c^1_{ik}$. For real-world situations, one would expect $c^0_{jk} + c^1_{jk} + c^2_{jk} > c^3_{jk}$ and, therefore, T_1 is less costly than T_2.

Third, the tour $T_3 = (i, j, k, j, i, k, i)$ only makes sense when the edge $\{j, k\} \in E^3$ is covered by two separate services. It is more costly than T_1 if $c^0_{ij} + c^1_{jk} + c^2_{jk} + c^0_{ik} > c^3_{jk}$ (one would expect this inequality to hold). Consequently, this is an example where zigzagging is advantageous.

2 Integer Programming Model

An intuitive formulation of the WRPPZ uses decision variables according to the different possibilities of service and traversal for each edge. Hence, for each edge $\{i, j\} \in E^K$, $K \in \{0, 1, 2, 3\}$, we define $2K$ decision variables $x^k_{ij}, x^k_{ji} \in \mathbb{N}_0$ for $k \in \{0, \ldots, K\}$. The x^k_{ij} model how often the corresponding edge is serviced/traversed in the particular manner. In order to abbreviate the formulation of an integer programming model, we define the vector $x_{ij} = (x^0_{ij}, \ldots, x^K_{ij})^T$ for edges $e = \{i, j\} \in E^K$. The associated costs are $c_{ij} = (c^0_{ij}, \ldots, c^K_{ij})^T$ and, accordingly, $\mathbb{1}^T x_{ij} = \sum_{k=0}^K x^k_{ij}$. Moreover, let (S, T) be the set of edges in E with one endpoint in $S \subseteq V$ and one in $T \subseteq V$.

$$z_{WRPPZ} = \min \sum_{\{i,j\} \in E} (c^T_{ij} x_{ij} + c^T_{ji} x_{ji}) \tag{1}$$

subject to

$$x^1_{ij} + x^1_{ji} = 1 \quad \text{for all } \{i, j\} \in E^1 \cup E^2 \tag{2}$$

$$x^2_{ij} + x^2_{ji} = 1 \quad \text{for all } \{i, j\} \in E^2 \tag{3}$$

$$x^1_{ij} + x^1_{ji} + x^3_{ij} + x^3_{ji} = 1 \quad \text{for all } \{i, j\} \in E^3 \tag{4}$$

$$x^2_{ij} + x^2_{ji} + x^3_{ij} + x^3_{ji} = 1 \quad \text{for all } \{i, j\} \in E^3 \tag{5}$$

$$\sum_{\{i,j\} \in (\{i\}, V)} \mathbb{1}^T x_{ij} - \sum_{\{j,i\} \in (V, \{i\})} \mathbb{1}^T x_{ji} = 0 \quad \text{for all } i \in V \tag{6}$$

$$\sum_{\{i,j\} \in (S, V \setminus S)} \mathbb{1}^T (x_{ij} + x_{ji}) \geq 2 \quad \text{for all } \varnothing \neq S \subsetneq V \tag{7}$$

$$x^0_{ij}, x^0_{ji} \in \mathbb{N}_0 \quad \text{for all } \{i, j\} \in E^0 \tag{8}$$

$$x^1_{ij}, x^1_{ji} \in \{0, 1\} \quad \text{for all } \{i, j\} \in E^1 \cup E^2 \cup E^3 \tag{9}$$

$$x^2_{ij}, x^2_{ji} \in \{0, 1\} \quad \text{for all } \{i, j\} \in E^2 \cup E^3 \tag{10}$$

$$x^3_{ij}, x^3_{ji} \in \{0, 1\} \quad \text{for all } \{i, j\} \in E^3 \tag{11}$$

The objective (1) is to find a postman tour with minimum cost. Constraints (2) and (3) state that each edge $\{i, j\} \in E^1 \cup E^2$ has to be serviced once resp. twice, in either direction. Zigzag service or separate services in either direction are guaranteed by (4) and (5) for all edges in E^3. Flow conservation constraints are given by (6) and, in a rural context, the connectivity of the postman tour is enforced by constraints (7). It is well-known that in (7) one can restrict the sets S to be the union of one or several connected components of $(V, E^1 \cup E^2 \cup E^3)$ (see, e.g., Eiselt *et al.* 1995a).

Note that for the special case of a windy Chinese postman problem (i.e., all edges are of type E^1 and costs fulfill $c^0_{ij} = c^1_{ij}$) the formulation reduces to the one given by Grötschel and Win (1992) when variables are substituted by $x'_{ij} = x^0_{ij} + x^1_{ij} \in \mathbb{N}_0$ so that equation (2) becomes $x'_{ij} + x'_{ji} \geq 1$.

3 Transformation into TSP

We propose a transformation that first maps the WRPPZ to an asymmetric TSP (ATSP). In case the WRPPZ is not symmetric, the transformation of Jonker and Volgenant (1983) is then used to transform the resulting ATSP into a symmetric TSP (STSP). This section focuses on the first transformation only. The key idea of the transformation is to model all required edges $e \in E^1 \cup E^2 \cup E^3$ of the WRPPZ by two or four ATSP nodes. We describe the internal connections between nodes modeling a single WRPPZ edge $e \in E^1 \cup E^2 \cup E^3$ first and present the connection between different edges afterwards. Let $M > 0$ be a suitable large number. Edges $e = \{i, j\} \in E^1$ are modeled by two nodes i_e, j_e with distances according to the distance matrix

$$\begin{pmatrix} - & -M + c^1_{ij} \\ -M + c^1_{ji} & - \end{pmatrix}. \tag{12}$$

Edges $e \in E^2 \cup E^3$ are modeled by four nodes $i^1_e, j^1_e, i^2_e, j^2_e$ with distance matrix

$$\begin{pmatrix} - & -M + c^1_{ij} & 0 & M \\ -M + c^1_{ji} & - & \Delta_{ij} & 0 \\ 0 & \Delta_{ji} & - & -M + c^2_{ij} \\ M & 0 & -M + c^2_{ji} & - \end{pmatrix}, \tag{13}$$

where Δ_{ij} is defined as $\Delta_{ij} = M$ for edges $e \in E^2$ and $\Delta_{ij} = c^3_{ij} - c^1_{ij} - c^2_{ij}$ for edges $e \in E^3$. Hence, the corresponding ATSP-digraph has $m = 2|E^1| + 4|E^2 \cup E^3|$ nodes. Let $C' = (c'_{ij})$ be the shortest path distance according to the cost of traversal, i.e., in the digraph (V, A^0, c^0), where A^0 is the arc set that contains two arcs (i, j) and (j, i) for each edge $\{i, j\} \in E$. Finally, nodes $i_e, k_{e'}$ in the transformed graph that belong to different edges $e = \{i, j\}, e' = \{k, \ell\} \in E$ are connected by arcs $(i_e, k_{e'})$ and $(k_{e'}, i_e)$ with cost c'_{ik}, resp., c'_{ki}. The principle of the transformation is visualized in Figure 1.

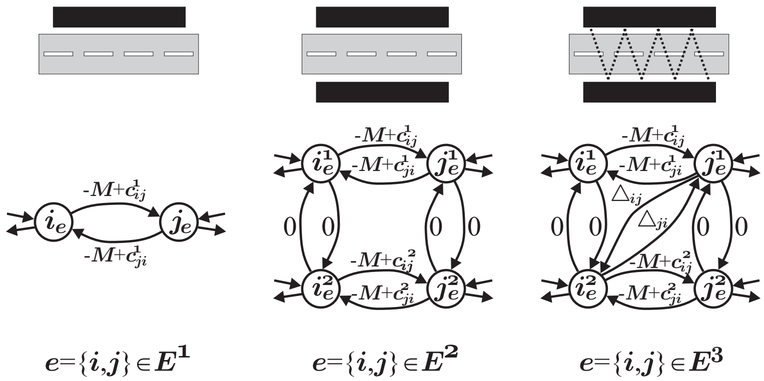

Fig. 1. Principle of the Transformation of WRPPZ into ATSP

The correctness of the transformation can be seen as follows. Any ATSP tour can only contain up to $|E^1|+2|E^2\cup E^3| = m/2$ times a coefficient $-M$. Hence, if M is large enough, an optimal ATSP tour connects all pairs (i_e, j_e), (i^1_e, j^1_e), resp., (i^2_e, j^2_e) of nodes belonging to the same required edge of the WRPPZ. Note further that any path $(i^1_e, j^1_e, i^2_e, j^2_e)$ for $e \in E^3$ corresponds with the zigzag service (in direction from i to j) and that its cost is $c^1_{ij} + \Delta_{ij} + c^2_{ij} - 2M = c^3_{ij} - 2M$.

The transformation presented here is similar to the transformations of arc routing problems into node routing problems given by Baldacci and Maniezzo (2004) for the capacitated arc routing problem (CARP), except that they do not consider zigzag services. Different transformations have been proposed by Laporte (1997) for postman problems into generalized ATSP (using one/two nodes for each directed/undirected required arc/edge) and Pearn *et al.* (1987) for CARP into capacitated VRP (using three nodes for each required edge).

4 Computational Results

Laporte (1997) and Blais and Laporte (2003) have shown that transformations of postman problems into TSP are not only appealing from a formal and modeling point of view, but that they work quite well even for large-scale problem instances. On the one hand, introducing big-M distances into the cost matrix certainly produces degenerate TSP instances. On the other hand, excellent heuristic and exact solution methods are available for the TSP, see e.g. Gutin and Punnen (2002).

The focus of the computational tests presented here is, therefore, not to discuss the quality of arc-into-node routing transformations in general. Instead, we will show that incorporating zigzag options into a model/algorithm has the potential of providing better solutions. An empirical computational test compares WRPPZ with WRPP. In order to make a fair comparison, we only consider WRPPZ instances where the cost of zigzag service is identical to the cost of the separate services, i.e., $c^3_{ij} = c^1_{ij} + c^2_{ij}$ for all $e = \{i,j\} \in E^3$. By changing all edges $e \in E^3$ into edges $e \in E^2$ a WRPP instance with the same costs but without zigzag option is created. Optimal solutions are computed using CONCORDE, a branch-and-cut STSP implementation provided by Applegate *et al.* (1999).

The random WRPPZ and WRPP instances were generated as follows. A sequence $x_1, \ldots, x_H$ of integers is generated by $x_1 := 0$, $x_{p+1} := x_p + U(30, 70), p = 2, \ldots, H$, where $U(30, 70)$ is a sample from the uniform distribution on $\{30, 31, \ldots, 70\}$. A second sequence $y_1, \ldots, y_H$ is generated in the same way. Each node $v \in V$ is located at one of the positions $(x_p, y_q), p, q \in \{1, \ldots, H\}$ in the Euclidean plane. In order to simulate a street network, only horizontal and vertical connections to the "neighboring" nodes define the edges $e \in E$, i.e., each node $v \in V$ is adjacent to up to four other nodes. A probability distribution $p = (p^0, p^1, p^2, p^3)$ controls the type of each edge $e \in E^0 \cup E^1 \cup E^2 \cup E^3$. Finally, all costs are symmetric and computed by the following rules. c^0_{ij} is the Euclidean distance between i and j. A single service is twice and zigzag service is four times as costly as a traversal, i.e., $c^0_{ij} = c^1_{ij}/2 = c^2_{ij}/2 = c^3_{ij}/4$.

Table 1 shows the results for ten different distributions p and sizes $H \in \{4, 5, \ldots, 9\}$ of rectangular street networks with H^2 nodes. Each entry in the table shows the aggregated computational results of 10 different randomly generated instances. The entry *%dev* is the *deviation* of the optimal objective between WRPPZ and the corresponding WRPP, i.e., $100 \cdot \frac{z_{WRPP} - z_{WRPPZ}}{z_{WRPPZ}}$. We present the average (avg), minimum (min), and maximum (max) deviation among the 10 instances of each block. Additionally, *# comp* is the *number of connected components* of the graph $(V, E^1 \cup E^2 \cup E^3))$ and *#nodes TSP* the *number of nodes* in the resulting TSP.

We interpret the results as follows. The deviation between WRPPZ and WRPP optimal solutions is correlated to the distribution $p = (p^0, p^1, p^2, p^3)$ of the edges. It does *not* significantly depend on the average portion of zigzag edges, i.e., p^3 or $p^3/(1 - p^0)$. For example, in distribution $p = (5, 1, 1, 3)/10$ the portion of zigzag edges is $\frac{3}{10}$, resp., $\frac{3}{5}$, while in distribution $p = (6, 2, 1, 1)/10$ the portion is $\frac{1}{10}$, resp., $\frac{1}{4}$, which is much smaller. Both distributions show a similar average deviation of about 2.25%. Similarly, the computational results show no correlation between the deviation and the number of connected components or between the deviation and the size of the resulting TSP.

It seems that the additional flexibility of zigzag service is most useful when only a small number of edges have to be serviced twice. The two distributions with a maximum value of $p^2 = 0.4$ give the smallest deviation, meaning that there is only a small potential for improvements when additional zigzagging is allowed. In addition, the five distributions with the highest improvement (average deviation) are those where the ratio of single and zigzag service edges compared to the double service edges, i.e. the quotient $(p^1 + p^3)/p^2$, is the highest. The more it is possible to service big parts of the network by a single walk through the edges, the higher is the gain of the additional flexibility to choose zigzag services.

5 Conclusions

The note has presented the WRPPZ as a generalization of postman problems with real-world applications in, e.g., waste-collection and postal services (Sniezek *et al.* 2002; Gendreau 2004; Voß 2004). The WRPPZ takes the option of a single zigzag service and two separate services into account. A transformation into an ATSP/STSP was given. It is straightforward to apply the same kind of transformation to any kind of capacitated arc-routing problems. From a theoretical point of view it is clear that the additional flexibility to build feasible postman tours (when zigzagging is an additional option) offers the potential of building better solutions. The computational tests indicate that improvements of some percent are possible. The size of the improvement depends on the cost of services but also on the distribution of the types of service edges.

Nowadays, the best metaheuristics for postman problems and CARP compete to close a small gap of a very few percent. In this context, improvements caused by the additional zigzag option, even below 1%, can be significant in some arc-routing applications. Obviously, one can expect more significant improvements when the cost of a zigzag service is smaller than the cost of two separate services. The additional flexibility of choosing between types of services could also be taken into account in a pre-processing by heuristically deciding on separate services or zigzag service. However, the simplicity of the transformation proposed in this note suggests that the decision on separate or zigzag services should be integrated into models and the corresponding solution algorithms.

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Table 1

Deviation between WRPPZ and WRPP Solutions, Number of Connected Components of the WRPP(Z), and Number of Nodes in the resulting TSP

Distribution p of the edges / Size	$(p^0,p^1,p^2,p^3) =$ $\frac{1}{10}(2,2,3,3)$	$\frac{1}{10}(3,1,4,2)$	$\frac{1}{10}(4,1,4,1)$	$\frac{1}{10}(5,1,2,2)$	$\frac{1}{10}(5,3,1,1)$	$\frac{1}{10}(5,1,1,3)$	$\frac{1}{10}(5,2,2,1)$	$\frac{1}{10}(5,2,1,2)$	$\frac{1}{10}(6,2,1,1)$	$\frac{1}{10}(6,1,2,1)$	Overall
$H=4$, $\lvert V\rvert=16$, $\lvert E\rvert=24$											
%dev avg/min/max	1,9/0/4,8	0,1/0/1,3	0,6/0/2,1	0,7/0/3,4	2,3/0/6,3	1/0/5	0,6/0/2,9	3/0/7	1,4/0/7,5	1/0/3,7	1,3/0/7,5
#comp avg	1,1	1,3	1,8	1,4	1,7	1,8	1,8	1,9	2,3	2,7	1,8
#nodes TSP avg	52,8	43,6	38,0	31,2	28,4	26,0	25,4	26,4	21,0	21,6	31,4
$H=5$, $\lvert V\rvert=25$, $\lvert E\rvert=40$											
	2,3/0,5/3,7	0,4/0/1,6	0,1/0/1	1/0/4,7	1,4/0/3,9	2/0/4,3	1,5/0/4,6	2,3/0,1/4,8	3,4/0/6,5	1,5/0/4,6	1,6/0/6,5
	1,0	1,4	1,7	2,5	3,3	2,4	2,5	2,6	4,0	4,1	2,6
	82,2	82,6	70,6	60,4	41,8	56,0	47,0	48,4	34,4	38,0	56,1
$H=6$, $\lvert V\rvert=36$, $\lvert E\rvert=60$											
	2,2/0,9/3,7	0,4/0/1,6	0,5/0/1,2	1,9/0,5/4,6	2,2/0/3,4	2,2/0/4,9	1,5/0,8/2,1	3,3/0,8/4,9	1,9/0/4,7	1,1/0/3,6	1,7/0/4,9
	1,0	1,2	1,7	3,1	3,8	3,5	4,2	4,4	5,6	4,6	3,3
	143,8	132,0	108,8	93,2	73,4	93,2	72,2	76,0	60,2	63,2	91,6
$H=7$, $\lvert V\rvert=49$, $\lvert E\rvert=84$											
	2,3/1,5/3,8	0,6/0/2,5	0,5/0/1,8	1,6/0/3,3	2/0/4,3	3,4/2,7/5,4	1,4/0/2,6	3,2/1/5,6	2,5/0,2/4,9	1,3/0/2,8	1,9/0/5,6
	1,1	1,4	2,5	4,1	5,6	4,8	4,8	4,7	7,0	6,9	4,3
	203,2	186,2	153,6	134,8	91,4	122,8	115,2	110,0	80,8	99,2	129,7
$H=8$, $\lvert V\rvert=64$, $\lvert E\rvert=112$											
	2,3/1,1/3,2	0,5/0/1,2	0,6/0/1,4	1,9/0,6/3,4	1,7/0,1/3,2	2,4/1,3/3,4	1,5/0,6/2,4	3,1/1,5/4,4	2,2/0,9/3,4	1,7/0,3/2,5	1,8/0/4,4
	1,2	1,7	3,1	6,5	6,1	4,5	5,9	4,6	8,5	9,1	5,1
	271,8	251,2	221,2	171,4	126,8	178,6	151,2	160,6	119,6	130,6	178,3
Overall											
%dev avg/min/max	2,2/0/4,8	0,4/0/2,5	0,5/0/2,1	1,4/0/4,7	1,9/0/6,3	2,2/0/5,4	1,3/0/4,6	3/0/7	2,3/0/7,5	1,3/0/4,6	1,7/0/7,5
#comp avg	1,1	1,4	2,2	3,5	4,1	3,4	3,8	3,6	5,5	5,5	3,4