The Additive Integrality Gap of the Skiving Stock Problem

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Abstract

The skiving stock problem (SSP) plays an important role whenever an efficient and sustainable use of limited resources is intended: find the maximum number of large objects (each satisfying a minimum threshold length) that can be obtained by recomposing a given inventory of small items. Besides presenting different ILP formulations for the SSP, the main part of this presentation deals with the additive integrality gap, i.e., the difference between the LP bound and the optimal value of the original NP-hard integer problem. In addition to its theoretical relevance, the quality of these LP bounds is a crucial factor in the size of branch-and-bound trees required to find an exact solution of the SSP. Although all available computations suggest that the gap is bounded by two, a holistic investigation of this property is very difficult. Hence, we first address an important subclass of instances (the divisible case) and present three new heuristics to efficiently construct integer solutions, whose objective values differ less than 1.5 units from the LP bound. Moreover, we provide strong evidence that this value can actually be reduced to 22/21. Based on these special-case considerations and further approaches, improved upper bounds for the gap of arbitrary instances are developed. Finally, instances possessing the currently largest known gaps are constructed to evaluate the tightness of the obtained upper bounds.