## Title: Cycles, Pricing, and Pivots

## Abstract:

It is known that all directed cycles necessary to reach an optimal network flow solution are observed on the so-called residual network. Each of these accommodates positive flow values and forms a direction. A degenerate pivot is induced when the selected cycle in fact does not exist. The concepts of cycles can be transferred to linear programs and alternative necessary and sufficient optimality conditions are expressed on the linear programming residual problem. We propose a family of algorithms with non-degenerate pivots and also show that the local search heuristics for vehicle routing problems, such as 2-opt, 3-opt, swap, relocate... are indeed directed cycles on the (appropriate!) residual network.