Modeling and Exact Solution of Picker Routing and Order Batching Problems

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Abstract

We present a new modeling and solution approach for two-level problems in warehousing where one level concerns picking operations in a manual picker-to-parts warehouse. In particular, we consider the single picker routing problem with scattered storage (SPRP-SS) and the joint order batching and picker routing problem (JOBPRP). The SPRP-SS assumes that an article is, in general, stored at more than one pick position. The task is then the simultaneous selection of pick positions for requested articles and the determination of a minimum-length picker tour collecting the articles. In the JOBPRP, a set of orders is given, each with one or several order lines requesting a number of articles. The problem is here to group the given orders into capacity-feasible batches so that the total length of the picker tours collecting the respective articles is minimized. It is a classical result of Ratliff and Rosenthal that, for given pick positions, an optimal picker tour is a shortest path in the state space of a dynamic program with a linear number of states and transitions. We extend the state space of Ratliff and Rosenthal so that every feasible picker tour is still a path. Furthermore, the additional requirement to make consistent selections and grouping decisions can be modeled as additional constraints in shortest-path problems. We propose to solve these problems with a MIP solver. We will explain why this approach is not only convenient and elegant but also generic: it covers optimal solutions to integrated problems that use heuristic routing policies for the picker tours, consider different warehouse layouts, and incorporate further extensions. Computational experiments with a direct MIP solver-based approach for the SPRP-SS and a branch-price-and-cut algorithm for the JOBPRP show that the new modeling and solution approach outperforms the available exact algorithms. The latter computes hundreds of new best and provably optimal solutions to open instances of three JOBPRP benchmark sets.

Keywords: warehousing, picker routing, order batching, scattered storage, branch-price-and-cut

1. Introduction

Warehouse activities include receiving, storing, picking, packing, and shipping operations (Gu *et al.*, 2007). Excellent surveys introduce warehouse operations planning including storage assignment, warehouse layout planning, zoning, routing, and batching (van Gils *et al.*, 2018; Boysen *et al.*, 2019). In this work, we address picking operations in manual (non-automated) warehouses, where pickers travel through the warehouse in order to collect articles from the storage locations (picker-to-parts). De Koster *et al.* (2007) highlight that more than 80% of all order-picking systems in Western Europe are low-level picker-to-parts picking systems. Order picking denotes the process of retrieving inventory items (articles) from their storage locations in response to specific customer requests (de Koster *et al.*, 2007; Masae *et al.*, 2020b). Manual order picking is certainly very labor-intensive and the literature gives different estimations for the effort: Often cited is the study of Tompkins *et al.* (2003) saying that typically 60% of all labor activities in the warehouse result from order picking and its cost can be estimated to be as much as 55% of the total

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warehouse operating expense. Frazelle (2002) estimates that order picking contributes to up to 50% of the total warehouse operating costs. These figures explain why research on order picking operations is extensive and of high practical relevance.

Many works have considered picker routing in isolation as it constitutes a well-defined subproblem in warehouse operations management. In its pure form, the *single picker routing problem* (SPRP) seeks a minimum-length picker tour given the warehouse layout and the pick positions from where articles must be collected. The SPRP can be considered solved, on the one hand exactly as shown in the seminal work of Ratliff and Rosenthal (1983) assuming a parallel-aisle single-block warehouse and on the other hand practically with routing policies which are rule-based heuristics such as traversal (a.k.a. S-shape), midpoint, largest gap (Hall, 1993), return, composite (Petersen, 1997), and combined (Roodbergen and de Koster, 2001a). The application of heuristic policies is well justified in settings where pickers cannot perform all types of optimal tours, which can be complicated, counter-intuitive, and difficult to memorize. Instead, pickers perform tours defined by some simple rules. Both exact and heuristic techniques have been extended into many different directions, e.g., to other warehouse layouts (Roodbergen and Koster, 2001; Ömer Öztürkoğlu *et al.*, 2012; Çelk and Süral, 2014), non-identical start and endpoints (Masae *et al.*, 2020a; Löffler *et al.*, 2022), and multiple end depots (de Koster and van der Poort, 1998; Goeke and Schneider, 2021).

We consider two integrated operational planning problems characterized by two levels of decisions. In both cases, the picker routing constitutes the lower-level decision, i.e., the lengths of different picker tours must be computed to evaluate higher-level decisions. The first example is the *SPRP with scattered storage* (SPRP-SS). When one or several articles are pickable from more than one pick position, the warehouse operates as a *scattered storage* warehouse or *mixed shelves* warehouse. Recent works (Weidinger, 2018; Boysen *et al.*, 2019; Weidinger *et al.*, 2019) stress that scattered storage is predominant in modern e-commerce warehouses of companies like Amazon or Zalando. The main advantage of this storage strategy is "that items of demanded SKUs are found close by irrespective of the position within the warehouse [so that] the distance to be covered for order picking is reduced this way" (Weidinger, 2018, p. 139). In the SPRP-SS, the higher-level decision is, for each requested article, the selection of one or several storage positions from where a sufficient number of this article can be collected. If the selection has been made, the resulting picker routing problem is the SPRP. However, both levels are interdependent and the SPRP-SS is known to be NP-hard (Weidinger, 2018), even if optimal routing is replaced by a heuristic routing policy (Korbacher *et al.*, 2022).

The second integrated problem is the *joint order batching and picker routing problem* (JOBPRP) (Gademann and van de Velde, 2005; Henn *et al.*, 2012; Valle *et al.*, 2017). Gademann and van de Velde distinguish between *proximity batching* (minimize the total length of the picker tours) and *time window batching* (minimize the sum of earliness and tardiness penalties). We consider proximity batching only, which is the objective that has been investigated in the vast majority of the works. Here, a feasible batch is one that the picker can carry, i.e., each order has a weight (physical weight or volume) and the accumulated weight of the orders in the batch must not exceed the given picker capacity. While the total weight of a batch is a separable function of its orders, the length of the picker tour is not. As a result, the higher-level decision on forming batches is interdependent with the computation of minimum-length picker tours. The JOBPRP is NP-hard except for batches that include not more than two orders (Gademann and van de Velde, 2005, Sect. 2 and Appendix).

1.1. Contributions

The focus of our work is on algorithmic improvements for exactly solving the SPRP-SS and JOBPRP. The effective solution algorithms we propose rely on the following underlying modeling approach: Every feasible picker tour is a path in the state space of the dynamic programming approach of Ratliff and Rosenthal (1983) and vice versa. The underlying assumption is that all pick positions are known and given. However, when picker routing problems are subproblems of integrated operational planning problems, the assembly of favorable pick lists creates a new situation in which the *selection* of orders or pick positions becomes essential. Our leading idea is to extend the state space of Ratliff and Rosenthal so that the selection aspect is fully modeled. In the extended state space, every feasible picker tour is still a path. The requirement to make consistent selections can be modeled as additional constraints in shortest-path problems. We show that these problems can be solved well as binary programs with the help of established *mixed-integer (linear) programming* (MIP) solvers.

The contributions of the paper at hand can be summarized as follows:

- Based on the idea of extending the state space of Ratliff and Rosenthal, we present new binary formulations for the SPRP-SS and JOBPRP. These are standard network-flow formulations of the origin-to-destination shortest-path problem (Ahuja *et al.*, 1993) complemented with additional constraints. The formulations are generic in the sense that they apply also to different warehouse layouts and other above-mentioned extensions, since dynamic-programming approaches are known for their solution.
- For the SPRP-SS, the direct MIP-based solution approach outperforms the very recent approach of Goeke and Schneider (2021), which uses another MIP formulation to be solved with a MIP solver, too. Note that their model is less general and restricted to the single-block parallel-aisle warehouse layout.
- For the JOBPRP, a Dantzig-Wolfe decomposition of the extended network-flow formulation enables us to solve the JOBPRP with a new type of *branch-price-and-cut* (BPC) algorithm, in which the column-generation procedure uses another network-flow formulation of a profitable SPRP solved again with a MIP solver. The BPC algorithm is the first column generation-based approach that guarantees, at the same time, optimal batches and optimal picker routes for the optimal policy for a general setting (previous approaches used heuristic policies only, considered the unit-weight case only, or were not fully-fledged BPC algorithms).
- An extensive computational study evaluates the performance of the BPC algorithm. The only alternative methods that guarantee both optimal batches and optimal picker tours in a general setting are MIP solver-based (Bahçeci and Öncan, 2021; Valle *et al.*, 2017). Our BPC algorithm is solving the entire JOBPRP benchmark Bahçeci and Öncan in less than one second per instance (on average), i.e., approximately two orders of magnitude faster than other approaches.
- JOBPRPs of realistic size are highly computationally challenging and have been mainly tackled with metaheuristics (Henn *et al.*, 2012; Žulj *et al.*, 2018). In these metaheuristics, picker routing using policies is predominant. Using the results of the companion work (Korbacher *et al.*, 2022), our BPC algorithm can also cope with heuristic routing policies, because heuristic picker routes can be computed over a restricted and slightly modified version of the extended state spaces. A large share of the standard benchmark of Henn and Wäscher (2012) only treated by metaheuristics yet can be solved to optimality now.
- In total, our new BPC algorithm for the JOBPRP computes hundreds of new best and provably optimal solutions to open instances of three JOBPRP benchmark sets.

1.2. Structure

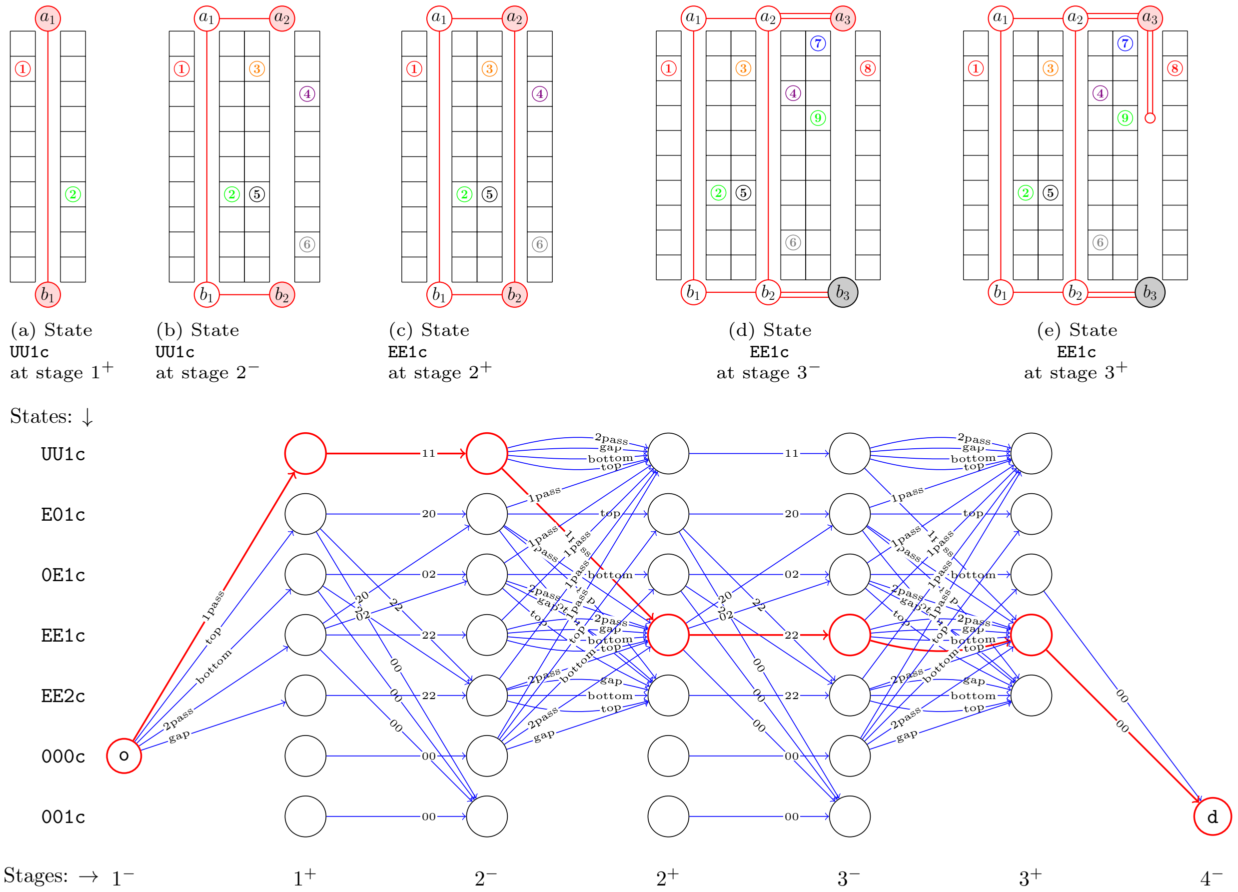
The paper is structured as follows. In Section 2, we review the dynamic-programming solution methods of Ratliff and Rosenthal (1983) and present models for the SPRP-SS and a profitable SPRP each using an extended state space. We formally introduce the JOBPRP in Section 3 and present a new compact binary formulation for it, from which a path-based formulation is derived via Dantzig-Wolfe decomposition. The latter formulation serves as the basis of our BPC algorithm for the JOBPRP that we detail in Section 4. Computational results for the SPRP-SS and JOBPRP are then reported in Section 5. Section 6 draws final conclusions and discusses future research directions.

2. Single Picker Routing Problems

In this section, we formalize the extended state spaces which are specific for different types of SPRP variants. Starting from the formal definition of the basic SPRP and a review of the dynamic-programming algorithm of Ratliff and Rosenthal (1983) in Section 2.1, we present the extended state spaces for the SPRP-SS (Section 2.2) and a profitable SPRP (Section 2.3). The latter is interesting in itself and will turn out being the pricing subproblem for the JOBPRP. In all cases, additional state transitions model picking activities not used in the algorithm of Ratliff and Rosenthal.

2.1. The Basic Single Picker Routing Problem and its State Space

The basic SPRP assumes that a set P of pick positions in the warehouse is given. The task is to find a minimum length picker tour that starts and ends at the given I/O point (depot) 0 and traverses all positions P. Clearly, this problem can be transformed into a *traveling salesman problem* (TSP) over the vertex set $P \cup \{0\}$, where the distance d_{ij} between a pair of vertices $i, j \in P \cup \{0\}$ is calculated according to the warehouse geometry. Such an approach is very generic because it allows arbitrary warehouse layouts (Theys *et al.*, 2010). On the downside, however, a general TSP algorithm does not consider the specific warehousing situation and is typically not efficient as the TSP is an NP-hard problem (Gutin and Punnen, 2002).



(a) State UU1c at stage 1^+ (b) State UU1c at stage 2^- (c) State EE1c at stage 2^+ (d) State EE1c at stage 3^- (e) State EE1c at stage 3^+

(f) State space

Figure 1: Example of a warehouse with $m = 3$ aisles, $C = 10$ cells per aisle, and articles to be collected, i.e., SKUs $S = \{1, 2, \ldots, 9\}$; (1a)–(1e) picker tour subgraphs of the optimal picker tour; (1f) state space of the dynamic program and optimal sequence of states and transitions (in red/thick).

More efficient solution approaches can exploit the warehouse layout. Starting with the seminal paper of Ratliff and Rosenthal (1983) for a standard parallel-aisle single-block warehouse, picker tours have be constructed in an aisle-by-aisle fashion giving rise to a *dynamic programming* (DP) formulation.

We briefly summarize the fundamental assumptions and ideas. In a single-block rectangular warehouse, the *stock keeping units* (SKUs) are stored in racks along both sides of several parallel (picking) aisles. (The terms *SKUs* and *articles* refer to the same objects, but we use article for what is requested by the customers, and SKU for the objects stored in the warehouse.) Cross aisles end the top and bottom of each aisle, they are storage-free. The same number of equidistant cells divides each of the picking aisles into pick positions. Picking from the right-hand side, from the left-hand side, or from both sides of a pick position are considered identical when computing picker tour distances. Hence, multiple picking requests with different SKUs from the identical pick position can be modeled as a single aggregated picking request. In the basic SPRP, the pick list boils down to a set of required pick positions that the picker needs to visit in his/her picker tour. The tour starts and ends at the given I/O point.

Figure 1 visualizes the construction of the optimal picker tour. The dynamic program models states and transitions over so-called *partial tour subgraphs* (PTSs) which are subgraphs of an undirected graph with vertices a_j and b_j located at the top and bottom of each aisle $j \in J$, respectively, where $J = \{1, 2, \ldots, m\}$ denotes the aisles set. One of these vertices also represents the depot. Moreover, additional vertices of the different PTSs are placed at those pick positions where the picker makes a U-turn. The states of the dynamic program represent the vertex parities (`0`, `U`, or `E` for not reached, odd (=uneven) degree, and even degree, respectively) of the vertices a_j and b_j. The PTSs comprise those parts of the picker tour that belong to the aisles 1 to j, either before the traversal of aisle j is included (we denote this stage as j^-) or after its inclusion (stage j^+). Moreover, the states also indicate the number of connected components of the PTS (`0c`, `1c`, or `2c` for the empty subgraph or a subgraph with one or two components, respectively). Ratliff and Rosenthal (1983) have shown that only seven states are possible for optimal picker tours, namely

$$\mathcal{S} = \{\texttt{UU1c}, \texttt{0E1c}, \texttt{E01c}, \texttt{EE1c}, \texttt{EE2c}, \texttt{000c}, \texttt{001c}\}.$$

States are connected in the DP according to two types of decisions: A transition from stage j^+ to $(j+1)^-$ determines a *cross-aisle traversal* from aisle j to the next aisle $j+1$, i.e.,

$$E_j^{cross} = \{\texttt{00}, \texttt{11}, \texttt{20}, \texttt{02}, \texttt{22}\},$$

where the first (second) digit gives the number of traversals of the top (bottom) cross aisle. A transition from stage j^- to j^+ determines the *aisle traversal*, i.e.,

$$E_j^{aisle} = \{\texttt{1pass}, \texttt{2pass}, \texttt{top}, \texttt{bottom}, \texttt{gap}, \texttt{void}\},$$

where `1pass` (`2pass`) stands for a single (double) traversal through the aisle (top to bottom or vice versa), `top` (`bottom`) for a traversal from the top (bottom) to the lowest (highest) pick position and back, `gap` for double-sided traversal from top and bottom leaving a maximum length gap in the middle, and `void` for no traversal through the aisle (the latter is only possible in aisles without products).

Each transition $e \in E_j^{cross} \cup E_j^{aisle}$ within an aisle $j \in J$ or between consecutive aisles j and $j+1$ is naturally associated with a cost describing the length of the part added to the picker tour. For example, the cost c_e for $e =$ `1pass` is (proportional to) the height H of the warehouse (top to bottom). Moreover, the cost c_e for $e =$ `gap` and aisle $j = 2$ in the example shown in Figure 1 is then $2H - 2v \cdot 4$, where v is the vertical distance between two neighboring cells in an aisle and the factor 4 results from the largest gap between SKUs 4 and 5 stored in aisle 2. Likewise, the cost c_e for $e =$ `22` is $4h$, where h is the horizontal distance between two neighboring aisles.

When solving the dynamic program, the cost 0 is assigned to the initial state `o` = `000c` at stage 0^- in a first step. Then, stage-by-stage (i.e., for $0^+, 1^-, 1^+, 2^-, \ldots$), the minimum cost is computed for all states $\sigma \in \mathcal{S}$ using the previously computed costs of states of the previous stage. Optimal decisions $e(\sigma) \in E_j^{cross} \cup E_j^{aisle}$ are stored for each $j \in J$. As a result, the cost attached to the final state $t =$ `001c` at stage $(m+1)^-$ is the length of a minimum-length picker tour. Backwards following the optimal decisions provides the optimal sequence of states and decisions. In Figure 1, the sequences are (`o` = `000c`, `UU1c`, `UU1c`, `EE1c`, `EE1c`, `EE1c`, `001c` = `d`) and (`1pass`, `11`, `1pass`, `22`, `top`, `00`) (highlighted in red/bold). A complexity analysis shows that the DP can be built-up and solved in $\mathcal{O}(m+n)$ space and time, when n is the number of articles to collect (Ratliff and Rosenthal, 1983; Heßler and Irnich, 2022).

The DP approach is not limited to parallel-aisle single-block warehouses. Indeed, for parallel-aisle warehouses with one or several 'middle' cross-aisles, Roodbergen and de Koster (2001a,b) have developed refined DP approaches. Instead of deciding traversals per aisle, the decision process is broken down into finer decisions per block and aisle. For example, with one middle cross-aisle there are three stages per aisle in the DP.

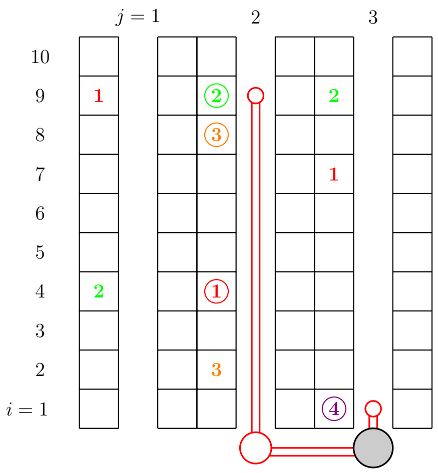
More generally, a fixed number of middle cross-aisles can be handled similarly so that the computational complexity of the DP remains linear in $m + n$ although the number of states grows considerably due to the numerous cases of connectivity. There exist several use cases where starting and ending points of the picker tour do not coincide, for which Masae *et al.* (2020a); Löffler *et al.* (2022) adapt the DP approach of Roodbergen and de Koster (2001b). DP-based optimal picker routing has also been suggested for different warehouse layouts such as the fishbone layout (the warehouse has two diagonal cross aisles from where picking aisles extend horizontally and vertically, see Çelk and Süral, 2014). The same authors proposed a graph transformation from the fishbone layout as well as the flying-V warehouse layout (the warehouse has parallel picking aisles perpendicular to the front and back cross aisles) to the rectangular layout with two blocks as considered by Roodbergen and de Koster (2001b).

What unifies all the DP models is that `o`-`d`-paths in the respective state space represent feasible picker tours and vice versa. The determination of an optimal picker tour is therefore equivalent to solving a shortest-path problem in an acyclic graph with only $\mathcal{O}(m)$ edges.

2.2. Scattered Storage and Extended State Space

Formalizing the SPRP-SS is straightforward. Let S denote the set of SKUs, i.e., different articles that appear in a given pick list. Each $s \in S$ is demanded d_s times where $d_s \in \mathbb{N}$ is a positive integer number. This demand can now be fulfilled by picks from one or several pick positions where the SKU is stored. To this end, let b_{sp} be the supply, i.e., the number of units of the SKU $s \in S$ available at position $p \in P$. If the picker tour visits a subset $P' \subseteq P$ of all pick positions, the demand is fulfilled whenever $\sum_{p \in P'} b_{sp} \geq d_s$ holds for all $s \in S$. In this case the picker tour is *feasible* for the SPRP-SS.

The *unit-demand* case of the SPRP-SS assumes $d_s = 1$ for all $s \in S$ and, therefore, also binary supplies $b_{sp} \in \{0, 1\}$ for all $s \in S$ and all positions p can be assumed. The unit-demand SPRP-SS can be modeled and solved as a *generalized TSP* (GTSP, Fischetti *et al.*, 2002). Indeed, the sets $P_s = \{(s, p) \in S \times P : b_{sp} = 1\}$ for $s \in S$ are the clusters. The start- and endpoint 0 of the picker tour at the I/O point can be included as an extra singleton cluster. The tasks of the GTSP is then to find a cost-minimal subtour that visits exactly one vertex from each cluster.



(a) The warehouse and optimal picker tour.

Aisle	Type of	additional aisle traversals
$j = 1$	`top`(i)	Cell $i = 9$
	`bottom`(i)	Cell $i = 4$
	`void`	
$j = 2$	`top`(i)	Cells $i \in \{4, 8\}$
	`bottom`(i)	Cells $i \in \{2, 4, 8^*\}$
	`gap`(h, i)	Cells $(h, i) \in \{(2, 8)^\ddagger, (2, 9), (4, 9)\}$
$j = 3$	`bottom`(i)	Cell $i \in \{1, 7\}$
	`gap`(h, i)	Cells $(h, i) = (1, 9)$

(b) Additional aisle traversals compared to non-scattered storage. $*$: dominated by `bottom`(4). ‡: dominated by `gap`$(2, 9)$.

Figure 2: SPRP with scattered storage. The pick list contains four SKUs $S = \{1, 2, 3, 4\}$ (unit demand); pick operations are encircled.

Figure 2a visualizes the SPRP-SS: There are four SKUs $S = \{1, 2, 3, 4\}$ to be collected. We assume unit demands, i.e., $d_1 = \cdots = d_4 = 1$. Supplies are placed as shown in the figure, i.e., the first three SKUs are stored at two or three positions each, while SKU 4 has a unique position in aisle 3. The depicted tour is feasible as it can collect all four SKUs.

In general, finding a shortest feasible picker tour in a parallel-aisle single block warehouse is strongly NP-hard, which has been proved by Weidinger (2018, Theorem 1). He also presents heuristics for the SPRP-SS. For the exact solution of the SPRP-SS with general demand, the only two evaluated solution approaches are MIP solver-based (Weidinger, 2018; Goeke and Schneider, 2021). We briefly summarize the model of Goeke and Schneider because we will compare their model (denoted as the GS-model in the following) with our new model presented below. The GS-model has some similarities with models known from arc routing (Corberán and Laporte, 2014). Instead of computing a sequence of consecutively visited positions, the solution of the GS model provides an Eulerian graph (connected and even), for which a picker tour can be computed afterwards (Eulerian graph first–routing second). Note that also the dynamic program of Ratliff and Rosenthal provides an Eulerian graph only. The main difference between transitions in the state space of Ratliff and Rosenthal and variables in the GS-model is that decisions in the GS-model are finer than decisions on consecutive PTSs. For example, the DP transition decides simultaneously on the traversal of the top and bottom cross aisle, while two decisions are made in the GS-model, one for the top and one for the bottom cross aisle. In the GS-model, the finer decision variables are then coupled by additional constraints ensuring consistency regarding even vertex degrees and connectivity.

It should also be noted that the GS-model approach outperforms the previous approach of Weidinger (2018) by at least one order of magnitude regarding MIP solver computation times and the size of warehouses that can be treated.

Extended State Space. We describe now the extended state space for a standard warehouse with parallel aisles and a single block layout. The number of stages and the states within each stage remain identical to the state space of Ratliff and Rosenthal. Moreover, cross-aisle traversals E_j^{cross} (connecting stages j^+ and $(j+1)^-$) remain identical. However, additional aisle traversals have to be added.

For the SPRP-SS depicted in Figure 2a, these are listed in Figure 2b. In aisle $j=1$, it is possible to pick only one SKU and also to completely skip this aisle because the SKUs 1 and 2 are also available in other aisles. In aisle $j=2$, SKU 3 must be collected, either from position $i=2$ or $i=8$ making `void` and `top`(9) impossible (the latter is the traversal from the top with a U-turn in cell $i=9$). Moreover, both `gap`(2,8) and `gap`(2,9) can collect SKUs 2 and 3 but not 1. For the last aisle $j=3$, SKU 4 must be collected so that `void`, `top`(7), and `top`(9) are not allowed. Note that all traversals `gap`(i,k) can be disregarded if i and k are neighboring positions with a non-maximal gap. As in the dynamic program of Ratliff and Rosenthal, these traversals are dominated by one with maximum gap. Overall, the set of aisle traversals becomes aisle dependent. Hence, we denote by E_j^{aisle} the resulting set of arcs connecting stages j^- and j^+ for all $j \in J$.

The aisle traversal set can be reduced by dominance considerations. In the example, `gap`(2,8) is dominated by `gap`(2,9) because of its higher cost. Note that this is only true in the unit-demand case, because otherwise `gap`(2,8) can provide more items of SKU 3. Likewise, `bottom`(8) is dominated by `bottom`(4).

For scattered storage with arbitrary demand, any aisle traversal $e \in E_j^{aisle}$ that leaves out the positions $P'_j \subset P$ in aisle j is feasible if and only if

$$\sum_{(s,p)\in P_s:\, p\notin P'_j} b_{sp} \geq d_s \qquad \forall s \in S. \tag{1}$$

The term on the left-hand side is the total supply of the not left-out positions (it must be possible to construct a feasible solution with the reached positions and all other aisle traversals referring to different aisles).

For convenience, we define P_e as the set of positions covered by an aisle traversal $e \in E_j^{aisle}$ in aisle $j \in J$. Then, $b_{se} = \sum_{p\in P_e} b_{sp}$ is the quantity of SKU $s \in S$ that can be collected when traversing the aisle via e. We denote the edges with a non-negative supply of SKU $s \in S$ by E_s. Cross-aisle traversals have zero supply $b_{se} = 0$ for all $e \in E_j^{cross}$ and $j \in J$.

For the following analysis of the size of the network and the model, it is important to properly distinguish between the set of relevant pick positions and all pairs (s,p) with a positive supply $b_{sp} > 0$. Note that several articles $s \in S$ might be available at the same pick position p, since, e.g., different SKUs might be placed to the left- and right-hand-side of one position. Moreover, when stored in shelves, different SKUs can even be placed vertically over one another at the same side. Summarizing, the number $n = |\bigcup_{s\in S} P_s| = |\{p \in P : \exists s \in S$ with $b_{sp} > 0|$ of relevant positions can be smaller than the number $|\{(s,p) \in S \times P : b_{sp} > 0\}|$ of pairs with a positive supply b_{sp}.

For counting the number of edges, note first that the original state space of Ratliff and Rosenthal has only $\mathcal{O}(m)$ edges. Note also that the number of additional aisle transitions in the extended state space is dominated by those of type $\texttt{gap}(h,i)$, where h and i are two different pick positions within an aisle. The worst case is that the majority of the pick positions concentrates in one or very few aisles so that there can be $\mathcal{O}(n^2)$ edges for aisle transitions of type $\texttt{gap}(h,i)$. Therefore, the total number of edges is bounded by $\mathcal{O}(m+n^2)$.

Network-Flow Formulations. We first analyze the situation with a unique aisle for a SKU before we formalize the general case. Whenever a SKU $s \in S$ is available in only a unique aisle j (e.g., SKUs 3 and 4 in aisles 2 and 3, respectively, see Figure 2), the feasibility regarding demand fulfillment is completely ensured by condition (1). Every $e \in E_j^{aisle}$ already ensures that the demand d_s can be collected in aisle j. Even more, a shortest $\texttt{o}$-$\texttt{d}$-path over the extended state space provides an optimal picker tour. No constraints in addition to flow-conservation constraints are mandatory. Hence, defining variables $x_e \geq 0$ for all $E = \bigcup_{j\in J}(E_j^{aisle} \cup E_j^{cross})$, directly leads to the standard shortest-path model for the SPRP-SS with unique aisles per SKU:

$$\min \sum_{e\in E} c_e x_e \tag{2a}$$

$$\text{subject to} \quad \sum_{e\in\delta^+(\sigma)} x_e - \sum_{e\in\delta^-(\sigma)} x_e = \begin{cases} +1, & \text{if } \sigma = \texttt{o} \\ -1, & \text{if } \sigma = \texttt{d} \\ 0, & \text{otherwise} \end{cases} \qquad \forall \sigma \in \mathcal{S} \tag{2b}$$

$$x_e \geq 0 \qquad \forall e \in E \tag{2c}$$

The length of the resulting picker tour is minimized by (2a) with appropriately defined aisle-traversal costs c_e for all $e \in E$. Flow conservation is ensured via (2b), where, for any state $\sigma \in \mathcal{S}$, $\delta^+(\sigma)$ and $\delta^-(\sigma)$ denotes the set of arcs leaving and entering state σ, respectively. The domain of the flow variables is given by (2c). Note that the demand fulfillment is ensured by the definition of E, since all SKUs s are only available in one unique aisle j.

The flow-conservation constraints (2b) can be rewritten in more compact form as $\mathcal{N}\mathbf{x} = \mathbf{u}_\texttt{o} - \mathbf{u}_\texttt{d}$ using the incidence matrix $\mathcal{N}$ of the digraph $(\mathcal{S}, E)$, the vector $\mathbf{x} = (x_e)_{e\in E}$ of the x-variables, and unit vectors $\mathbf{u}_\texttt{o}$ and $\mathbf{u}_\texttt{d} \in \{0,1\}^{\mathcal{S}}$. The general model for a not necessarily unique aisle per SKU is:

$$\min \sum_{e\in E} c_e x_e \tag{3a}$$

$$\text{subject to} \quad \mathcal{N}\mathbf{x} = \mathbf{u}_\texttt{o} - \mathbf{u}_\texttt{d} \tag{3b}$$

$$\sum_{e\in E_s} b_{se} x_e \geq d_s \qquad \forall s \in S \tag{3c}$$

$$x_e \in \{0,1\} \qquad \forall e \in E \tag{3d}$$

The difference between models (2) and (3) is that for the general case, the flow variables must be forced to binary values due to the additional SKU-covering constraints (3c). The latter constraints can be strengthened by replacing b_{se} with $\min\{d_s, b_{se}\}$ for all $s \in S$. Moreover, SKU-covering constraints (3c) are redundant for every SKU s stored in a unique aisle. We assume both refinements in the following.

We define $a = |S|$ as the *number of different articles*. A straightforward analysis of the size of formulation (3) can be summarized as follows: The number of variables coincides with the number of edges in the extended state space which is bounded by $\mathcal{O}(m+n^2)$. The number of constraints is bounded by $\mathcal{O}(m+a)$, where constraints (3b) contribute the summand m and constraints (3c) the summand a. The number of non-zero coefficients is bounded by $\mathcal{O}(m+an^2)$, since the incidence matrix has exactly $2|E| = \mathcal{O}(m+n^2)$ non-zeros, and the SKU-covering constraints (3c) have $\sum_{s\in S}|E_s| = \mathcal{O}(an^2)$ non-zero coefficients.

Clearly, the network-flow model (3) hides complicated details in the definition of the underlying state space over which the flow-conservation constraints are defined. The model is generic in the sense that other warehouse layouts could be combined with scattered storage in the same way by extending their original state spaces (Roodbergen and de Koster, 2001b,a; Ömer Öztürkoğlu *et al.*, 2012; Çelk and Süral, 2014). Moreover, not only optimal picker tours but also distance-minimal tours for routing policies can be modeled with slightly modified state spaces (Korbacher *et al.*, 2022).

2.3. Profitable Single Picker Routing and Extended State Space

In this subsection, we assume that in addition to the warehouse layout, a set of orders is given each comprising one or several order lines with requested quantities of articles. All articles of an order must be collected together in one picker tour. In the *profitable SPRP*, each order has a given profit and weight. The task is to select a feasible subset of orders such that the difference between the profit generated from the selected orders and the routing cost of a picker tour collecting the orders is maximized. A selection of orders is feasible if the total weight of the selected orders does not exceed the given capacity of the picker. The profitable SPRP has not been studied before in isolation. However, it has been considered as the subproblem in JOBPRPs, see Sections 3 and 4.

Formally, let O denote the set of *orders*. Each order $o \in O$ is characterized by a non-negative profit π_o and a subset $S_o \subset S$ of articles requested in the customer's order. As the order also specifies the quantity of each article, one can define a weight $q_o > 0$ of the order (in kg, liter, or the number of compartments required to place it on the picking cart). A subset of orders is *feasible* if its total weight does not exceed the picker capacity Q. In contrast to the SPRP-SS, it is assumed that every pick position of a SKU holds sufficiently many items to fulfill the demand induced by any feasible selections of orders. This simplification means that orders do not compete for the available SKUs.

Extended State Space. As for the SPRP-SS, additional aisle traversals must be included into the sets E_j^{aisle} for all $j \in J$ compared to both the basic SPRP and the SPRP-SS. Clearly, when orders can be selected, we do no longer know that specific articles are collected. Consequently, for all aisles $j \in J$, traversal `void` as well as traversals `top`(i) and `bottom`(i) must be considered for all positions i storing a relevant SKU. Even more, additional aisle traversals `gap`(h, i) must be considered for all pairs (h, i) of non-consecutive positions h and i with $h < i$. Note that also this extension is aisle-dependent.

As before, the extended sets E_j^{aisle} can be reduced. For example, if a traversal does not collect one SKU s in aisle j then all orders containing s cannot be fulfilled. If some of these orders uniquely contain additional SKUs in aisle j, collecting these SKUs is needless. The corresponding positions h and i in `top`(i), `bottom`(i), and `gap`(h, i) can be disregarded.

The extended state space has $\mathcal{O}(m)$ states and $\mathcal{O}(m + a^2)$ edges in the worst case.

Network-Flow Formulations. The new model for the profitable SPRP extends the network-flow formulation (5) by two types of decisions: Binary variables z_o for all orders $o \in O$ describe the order-selection aspect. Moreover, the model uses auxiliary SKU-indicator binary variables y_s for all $s \in S$.

$$c(\boldsymbol{\pi}) = \min \sum_{e \in E} c_e x_e - \sum_{o \in O} \pi_o z_o \tag{4a}$$

$$\text{subject to} \quad \mathcal{N}\mathbf{x} = \mathbf{u}_{\mathsf{o}} - \mathbf{u}_{\mathsf{d}} \tag{4b}$$

$$\sum_{e \in E_s} x_e \geq y_s \qquad \forall s \in S \tag{4c}$$

$$y_s \geq z_o \qquad \forall o \in O, \forall s \in S_o \tag{4d}$$

$$\sum_{o \in O} q_o z_o \leq Q \tag{4e}$$

$$x_e \in \{0, 1\} \qquad \forall e \in E \tag{4f}$$

$$y_s \in \{0, 1\} \qquad \forall s \in S \tag{4g}$$

$$z_o \in \{0, 1\} \qquad \forall o \in O \tag{4h}$$

The objective (4a) minimizes the picker tour length minus the collected profit, which gives the maximum difference between the two values. As before, flow conservation is ensured by constraints (4b). Similar to the demand-fulfillment constraints (3c), the constraints (4c) enforce that the picker tour visits all necessary SKU positions in the warehouse. The decision of which SKUs to collect result from selected orders in the coupling constraints (4d). The capacity constraint (4e) guarantees the feasibility of the chosen subset of orders. The domains of all variables are stated in (4f)–(4h).

Note that our naming 'profitable SPRP' is consistent with the pertinent literature, since constraints (4b) clearly characterize it as a picker-routing problem, and the classification of Feillet *et al.* (2005) denotes as *profitable* those routing problems that simultaneously minimize routing costs and maximize profit resulting from the selected non-mandatory services. Moreover, one can see similarities with non-disjoint clustered routing problems because every selected order with more than just one order line defines a non-trivial cluster of positions to visit.

We briefly analyze the size of formulation (4): It has $\mathcal{O}\left(m + a^2 + |O|\right)$ variables, since $|E| \in \mathcal{O}\left(m + a^2\right)$ as shown above. The number of constraints is bounded by $\mathcal{O}\left(m + l\right)$ with $l = \sum_{o \in O} |S_o|$ giving the number of order lines. The last summand l is a result of the coupling constraints (4d).

3. Joint Order Batching and Picker Routing

The JOBPRP can be defined with similar inputs as the profitable SPRP: A JOBPRP instance is defined by the warehouse layout, the SKUs present in the warehouse, a routing policy, and a set O of orders each comprising a set of articles $S_o \subseteq S$ in different multiplicity resulting in a weight q_o for each order $o \in O$. Also here, all articles of an order must be collected together in one picker tour. A picker can collect several orders together in one tour if the weight of the orders does not exceed the capacity Q. Such a feasible set of orders is called a batch. The task of the JOBRPR is to partition the orders into batches such that the total length of the corresponding tours is minimized.

We briefly review exact algorithms for the JOBPRP and refer to (Žulj *et al.*, 2018) for an overview of (meta)heuristic algorithms. Valle *et al.* (2016) published the first exact JOBPRP algorithm for policy optimal. They present three different formulations, all with a polynomial number of binary x_{bij}-variables representing that the bth picker moves directly between two positions i and j, where a position is either a pick position or an auxiliary position located at some aisle crossing. Their first formulation has exponentially many generalized subtour breaking constraints, while the other two are compact using either one or multiple commodity flows per order to eliminate infeasible subtours. The authors also mention some limitations of their models: First, for batch-indexed models (see also Section 3.1), finding the 'exact [number of batches] required to service all orders is an optimisation problem on its own'. They set an upper bound for the number of batches in the solution a priori, which does not necessarily guaranty minimum-length JOBPRP solutions. Second, all articles are assumed having identical weight 'irrespective of their shapes and sizes'. The subsequent article (Valle *et al.*, 2017) of the same authors extends the branch-and-cut approach for a slight variation of the first formulation by deriving several families of valid inequalities. The improved branch-and-cut solves two-block warehouse instances with up to 20 orders to proven optimality. Interestingly, (Valle *et al.*, 2017, p. 833) suggest to develop a BPC algorithm in which the picker tour-generation subproblem should exploit the sparse nature of the warehouse network. In contrast, our approach exploits the sparsity of the Ratliff and Rosenthal network.

Bahçeci and Öncan (2021) present tailor-made MIP formulations for the JOBPRP with policies composite, mixed, largest gap, and optimal, resorting to (Öncan, 2015) for the policies traversal, return, and midpoint. Also these models can be characterized as batch-indexed. For the optimal policy, a commodity-flow formulation with variables $x^b_{ss'}$ is used, where b identifies the picker and s and s' are different SKUs collected consecutively. Corresponding flow variables ensure that all articles of an order are collected together. As far as we know, this model is the first that is general and optimal (jointly optimizing batches and picker tours using policy optimal). They solve instances with up to $|O| = 12$ orders, $l = 30$ order lines, and $\bar{b} = 6$ batches.

Moreover, Gademann and van de Velde (2005), Muter and Öncan (2015), and Briant *et al.* (2020) have applied column generation-based methods. They rely on a set-partitioning formulation of the JOBPRP, which is formally identical to the decomposition-based formulation that we present in Section 3.2. We will analyze and discuss their approaches in more detail in Section 4: none is both optimal and general (in the sense discussed above). However, all three approaches could be extended and would, therefore, give rise to exact JOBPRP algorithms.

Finally, the JOBPRP can also be modeled as *soft-clustered vehicle routing problem* (SoftCluVRP) (Aerts *et al.*, 2021). The SoftCluVRP is a variant of the *capacitated vehicle routing problem* (CVRP), in which customers are assigned to clusters and all customers of a cluster must be visited by the same vehicle, not necessarily consecutively. The analogy between JOBPRP and SoftCluVRP is that the order lines can be seen as the customers (their weight is the demand), the orders are the clusters, and the pickers are the available vehicles. The exact algorithms of Hintsch and Irnich (2020) and Heßler and Irnich (2021) for the SoftCluVRP, a branch-and-price and a branch-and-cut, work with arbitrary distances between customers (pick positions) and are therefore independent of the warehouse layout. On the downside, they do not at all exploit the specific layout.

3.1. Network-Flow Formulation

The new network-flow formulation of the JOBPRP can be characterized as a batch-indexed formulation. A prerequisite of these types of models is that an upper bound $\bar{b}$ for the number of picker tours in a solution can be specified in advance (we discuss this assumption below). Let $B = \{1, 2, \dots, \bar{b}\}$ be the index set of the batches that represent a possible solution. The extended state space for the JOBPRP is the same as the one for the profitable SPRP with the following addition: We add an edge from origin $\mathtt{o}$ to destination $\mathtt{d}$ with cost 0 to the extended state space, since $\bar{b}$ is an upper bound on the number of picker tours. Hence, we are able to also represent empty batches.

The network-flow formulation duplicates the variables adding the index $b \in B$ to them. Accordingly, binary flow variables x_e^b indicate whether transition $e \in E$ is part of the picker tour of batch $b \in B$. The binary variables y_s^b show whether article $s \in S$ is collected in batch $b \in B$. Furthermore, the binary variables z_o^b indicate whether order $o \in O$ is included in batch $b \in B$. The result is the following pure binary model:

$$\min \sum_{b\in B}\sum_{e\in E} c_e x_e^b \tag{5a}$$

$$\text{subject to} \quad \sum_{b\in B} z_o^b = 1 \qquad \forall o \in O \tag{5b}$$

$$\mathcal{N}\mathbf{x}^b = \mathbf{u}_\mathtt{o} - \mathbf{u}_\mathtt{d} \qquad \forall b \in B \tag{5c}$$

$$\sum_{e\in E_s} x_e^b \geq y_s^b \qquad \forall b \in B, \forall s \in S \tag{5d}$$

$$y_s^b \geq z_o^b \qquad \forall b \in B, \forall o \in O, \forall s \in S_o \tag{5e}$$

$$\sum_{o\in O} q_o z_o^b \leq Q \qquad \forall b \in B \tag{5f}$$

$$x_e^b \in \{0,1\} \qquad \forall b \in B, \forall e \in E \tag{5g}$$

$$y_s^b \in \{0,1\} \qquad \forall b \in B, \forall s \in S \tag{5h}$$

$$z_o^b \in \{0,1\} \qquad \forall b \in B, \forall o \in O \tag{5i}$$

The objective (5a) is the minimization of the total length of all picker tours. Constraints (5b) partition the orders into not more than $\bar{b}$ batches. The flow-conservation constraints (5c) guarantee a (possibly empty) picker tour for every batch index $b \in B$. The coupling between the flow variables x and the SKU-indicator variables y is accomplished via constraints (5d). Likewise, the coupling between the indicator y- and z-variables is accomplished by (5e). Inequalities (5f) are the capacity constraints. Finally, the variable domains are defined by (5g), (5h), and (5i).

Some further remarks about formulation (5) are due: Constraints (5c)–(5f) are $\bar{b}$ copies of the constraints (4b)–(4e) for the profitable SPRP. The new model for the JOBPRP has $\mathcal{O}\left(\bar{b}m + \bar{b}a^2\right)$ variables and

$\mathcal{O}(\bar{b}m + \bar{b}l)$ constraints where $l = \sum_{o \in O} |S_o|$ is the number of order lines. The relation $a \leq l$ holds true, but $a^2 \ll l$ or $a^2 \gg l$ is possible.

Finding a valid upper bound $\bar{b}$ for the number of picker tours is a non-trivial task. The minimum number $\underline{b}$ of tours can be computed as the solution of a bin-packing problem with capacity Q and weights $(q_o)_{o \in O}$. However, solving formulation (5) with $\bar{b}$ set to the minimum number $\underline{b}$ of tours does not guarantee a solution with minimum total length (same situation as for the CVRP). Pretests with formulation (5) have also revealed that setting $\bar{b}$ to a larger value than $\underline{b}$ leads to substantially longer MIP solver solution times. Overall, we do not further investigate a direct MIP solver-based solution approach but decompose formulation (5), as discussed in the following.

3.2. Path-Based Formulation

The network-flow formulation (5) has a block structure, i.e., all constraints except for (5b) decompose into groups by batch index $b \in B$. Therefore, a Dantzig-Wolfe decomposition according to the order partitioning conditions (5b) is natural (Lübbecke and Desrosiers, 2005). Since all blocks are identical with respect to $b \in B$, a subsequent aggregation (over $b \in B$) leads to a b-index-free formulation, which has the clear advantage of eliminating the inherent symmetry. For a formal derivation of the decomposed formulation, let $(\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}}) \in \{0,1\}^{|E|+a+|O|}$ be an extreme point of a block. Since all variables are binary, the set of these extreme points is

$$\mathcal{W}_0 = \{(\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}}) \in \{0,1\}^{|E|+a+|O|} : \text{fulfills (4b)–(4e)}\}.$$

Each extreme point $w \in \mathcal{W}_0$ is either $\mathbf{0}$ (for the empty picker tour) or represents a feasible picker tour together with the selection of orders collected by the tour. We can disregard the empty picker tour and define the non-zero extreme points as $\mathcal{W} = \mathcal{W}_0 \setminus \{\mathbf{0}\}$. This yields the following Dantzig-Wolfe (integer) master program a.k.a. extensive formulation (Lübbecke and Desrosiers, 2005):

$$\min \sum_{w=(\bar{\mathbf{x}},\bar{\mathbf{y}},\bar{\mathbf{x}}) \in \mathcal{W}} (\mathbf{c}^\top \bar{\mathbf{x}}) \lambda_w \tag{6a}$$

$$\text{subject to} \sum_{w=(\bar{\mathbf{x}},\bar{\mathbf{y}},\bar{\mathbf{z}}) \in \mathcal{W}} (\bar{z}_o) \lambda_w = 1 \qquad \text{dual: } [\pi_o] \qquad \forall o \in O \tag{6b}$$

$$\lambda_w \in \{0,1\} \qquad \forall w \in \mathcal{W} \tag{6c}$$

The objective function (6a) minimizes the total length of all tours $w \in \mathcal{W}$. The set-partitioning constraint (6b) ensure that each order $o \in O$ is collected by exactly one picker tour $w \in \mathcal{W}$. The variable domains are given by (6c). Note that the generalized convexity constraint $\sum_{w \in \mathcal{W}} \lambda_w = \bar{b}$ is redundant (omitted in the model), because $\bar{b}$ is assumed non-constraining and empty picker tours were allowed in the original formulation (5). It is a matter of taste whether one calls model (6) a set-partitioning, tour-based, or path-based formulation because a picker tour is a path in the extended Ratliff and Rosenthal state space.

4. Branch-Price-and-Cut

A BPC algorithm is a branch-and-bound algorithm in which in each node the linear relaxation of an extensive formulation is solved with column generation, and cutting planes are added to strengthen the linear relaxations (Desaulniers *et al.*, 2005). Accordingly, the following subsections describe the column-generation mechanism, the branching rule and tree search strategy, and the integration of the non-robust subset-row inequalities. Moreover, our BPC algorithm uses the MIP solver to find a feasible JOBPRP integer solution and tight upper bound as early as possible.

Note: A modern BPC algorithm typically has several algorithmic components that have to be parameterized. In the following, we describe the structure of the algorithmic components and use symbolic parameters most of the time. Section 5.3.1 then sketches the pretests performed for parameterization and provides concrete parameter values.

4.1. Column Generation

Column generation is used to solve the linear relaxation of a *restricted master program* (RMP), which comprises a subset of all variables only. For formulation (6), we initialize the RMP with artificial big-M variables. Let $\boldsymbol{\pi} = (\pi_o)_{o\in O}$ the dual prices of the partitioning constraints (6b) of an RMP. The task of each column-generation iteration is to identify at least one negative (preferably minimum) reduced cost variable or to prove that none exists. The reduced cost of a variable λ_w for an extreme point $w = (\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathcal{W}$ is

$$\tilde{c}_w(\boldsymbol{\pi}) = \mathbf{c}^\top \mathbf{x} - \pi^\top \mathbf{z} = \sum_{e\in E} c_e x_e - \sum_{o\in O} \pi_o z_o, \tag{7}$$

which is identical to the objective function value $c(\boldsymbol{\pi})$ of the SPRP with profits $\boldsymbol{\pi}$ stated in (4a). Hence, formulation (4) is the column-generation subproblem a.k.a. pricing problem.

4.1.1. Exact Pricing

The pricing problem (4) must be solved exactly to finish the column-generation process in each node of the branch-and-bound tree. If $c(\boldsymbol{\pi}) \geq 0$ is proven, the linear relaxation provides a valid lower bound.

We propose solving formulation (4) directly with the help of a MIP solver. Such an approach is rather unusual because problem-tailored algorithms are typically used in many other column-generation algorithms, e.g., DP labeling algorithms for solving *shortest-path problems with resource constraints* (SPPRCs) in vehicle routing and crew scheduling (Irnich and Desaulniers, 2005) and algorithms for variants of knapsack problems and in cutting and packing applications (Kellerer *et al.*, 2004). Since the JOBPRP can be modeled and solved as a SoftCluVRP (see the previous section), the direct MIP solver-based approach is in line with the analysis performed by Hintsch and Irnich (2020) showing that DP labeling is by far inferior to the MIP solver for solving SoftCluVRP pricing problems. Prior and independently from the SoftCluVRP, alternative JOBPRP-tailored pricing algorithms have been developed and used successfully for three other column generation-based algorithms. Table 1 provides a synopsis of the three algorithms that we review in the sequel.

Table 1: Column generation-based approaches for the JOBPRP

		JOBPRP			Methodology	
Reference and type of algorithm	Exact Alg.	Routing Policies	Order Weight	Layout	Column Generation	Features
(Gademann and van de Velde, 2005) Branch-and-price	yes	optimal	unit weight	single block	combinatorial branch-and-bound	Ryan and Foster branching
(Muter and Öncan, 2015) Column-and-row-generation, column enumeration, and MIP solver	yes	traversal, midpoint, return	general	single block	combinatorial branch-and-bound	subset-row inequalities; enumeration with reduced cost-based pruning
(Briant *et al.*, 2020) Column generation, MIP solver on RMP	no	optimal	general	two block	Benders-like cutting-plane algorithm	no branching implemented; sometimes incomplete pricing
Our BPC algorithm	yes	optimal, traversal, midpoint, return, largest gap, composite	general	single block	pricing heuristics; MIP solver for exact pricing	subset-row inequalities; Ryan and Foster branching

Branch-and-price algorithm by Gademann and van de Velde. Gademann and van de Velde (2005) assume that pickers use trolleys and that all requested articles of an order fit into one bag/compartment of a trolley. Since articles of any two orders are kept separate, each order $o \in O$ has unit weight $q_o = 1$. This assumption allows them to effectively solve the pricing problem to optimality with the help of a combinatorial branch-and-bound algorithm sketched briefly now.

All orders are first sorted by decreasing profit such that $\pi_1 \geq \pi_2 \geq \cdots \geq \pi_\ell$. Then, one by one, the branch-and-bound algorithm decides on the inclusion of the next order (in the given sorting; the branch-and-bound tree has a maximum height of Q levels). At level $k-1 \leq Q$, a partial solution $\{o_{i_1}, o_{i_2}, \ldots, o_{i_{k-1}}\}$ is given with $1 \leq i_1 < i_2 < \cdots < i_{k-1} \leq \ell$. The next order o_{i_k} to include into the pricing solution can then only be selected for an $i_k > i_{k-1}$. The new partial solution $O' = \{o_{i_1}, o_{i_2}, \ldots, o_{i_{k-1}}, o_{i_k}\}$ has an associated lower bound $LB(k, i_k) = c(O') - \sum_{o \in O'} \pi_o - \sum_{i=i_k+1}^{i_k+Q-k} \pi_{o_i}$, where $c(O')$ is the minimum length of a picker tour serving O'. The reasoning behind this lower bound is that (i) the final picker tour cannot be shorter than $c(O')$ (first term), (ii) the profit of the selected orders is known (second term), and (iii) not more than $Q-k$ additional orders can be selected from $\{o_{i_k+1}, o_{i_k+2}, \ldots, o_\ell\}$, where the most profitable of these are order o_{i_k+1} and its direct successors.

The largest JOBPRP instances solved with the BPC of Gademann and van de Velde (2005) have 32 orders, a capacity of $Q = 10$, and on average ten articles per order (some extreme instances have 20 articles per order but a smaller number of orders and capacity). The instances used in the article have not been published.

Algorithm by Muter and Öncan. The algorithm of Muter and Öncan (2015) follows a successful column generation-based approach first suggested and implemented by Baldacci *et al.* (2008). Instead of relying on a tailor-made branching in BPC, only the linear relaxation of the master program is solved, iteratively cuts are added to produce a very tight lower bound, all columns with reduced cost smaller than the integrality gap are enumerated using a reduced cost-based pruning technique, and finally the integer model with the enumerated columns is solved via MIP solver. A requirement is a tight upper bound, which can be computed with a metaheuristic or estimated and refined if needed.

For the column-generation part, Muter and Öncan generalize the idea of Gademann and van de Velde for the general weight case. Instead of sorting orders by profit, they are sorted by relative profit π_o/q_o. Moreover, the partial solutions are now characterized by their accumulated weight. Although the adapted combinatorial branch-and-bound would work for any policy, Muter and Öncan consider only traversal, midpoint, and return. Thus, not necessarily optimal picker tours are constructed for the more general case of integer order weights. As a consequence, results of the two works (Gademann and van de Velde, 2005) and (Muter and Öncan, 2015) cannot be properly compared. We compare our BPC algorithm with the results (Muter and Öncan, 2015) in Section 5.3.4.

Column-generation algorithm by Briant et al.. Briant *et al.* (2020) suggest to solve the pricing problem with a Benders-like cutting-plane algorithm. They formulate the pricing problem (4) with binary order indicator variables z_o for all $o \in O$ (the same as in our model (4)) and a single continues non-negative variable C that describes the length of the picker tour:

$$c(\pi) \quad = \quad \min \quad C - \sum_{o \in O} \pi_o z_o \tag{8a}$$

$$\text{subject to} \quad \text{(4e) and (4h)} \tag{8b}$$

$$c(O') \cdot \left(\sum_{o \in O'} z_o - |O'| + 1 \right) \leq C \qquad \forall \varnothing \neq O' \subset O \tag{8c}$$

$$C \geq 0 \tag{8d}$$

Objective (8a) minimizes the difference between picker tour length and the profit resulting from the selected orders. The so-called *tour constraints* (8c) establish the correct picker tour length depending on the selected orders, i.e., they couple the z-variables with the variable C: For orders O', the term in brackets is always non-positive except for the case that all orders in O' (or possibly more) are selected. In this case, the term is equal to one, and it pushes the value of C to at least the length $c(O')$ of the picker tour that serves the orders O'.

The advantage of having a small number of variables in model (8) comes at the cost of an exponential number of tour constraints. For solving formulation (8) effectively, a tailor-made solution approach is required. Briant *et al.* developed a Benders-like cutting-plane algorithm for this purpose, i.e., the model is solved iteratively with a subset of the tour constraints.

For the description of the method, we assume that only a subset of the tour constraints (8c) is present. Formally, let $\mathcal{S} \subset 2^O$ be a collection of subsets of orders, i.e., the quantifier $\forall \varnothing \neq O' \subset O$ is replaced by $\forall O' \in \mathcal{S}$ in (8c). The relaxed formulation using $\mathcal{S}$ only is the *Benders master problem* $BM(\mathcal{S})$. A possible starting point is $\mathcal{S} = \varnothing$ making the Benders master equivalent to the binary knapsack problem given by $C = 0$ and (8a)–(8b). Briant *et al.* initialize $\mathcal{S}$ with all subsets of orders that define the variables of the RMP.

A Benders iteration starts with an integer solution $(\hat{\mathbf{z}}, \hat{C})$ of the Benders master $BM(\mathcal{S})$. This solution is already a solution to the pricing problem if and only if $c(\hat{O}) \leq \hat{C}$ for the selected orders $\hat{O} = \{o \in O : \hat{z}_o = 1\}$. Otherwise, the tour constraint for $\hat{O}$ is violated. The process is iterated by extending $\mathcal{S}$ to $\mathcal{S} \cup \{\hat{O}\}$ and re-optimizing $BM(\mathcal{S})$. Note that the check whether $c(\hat{O}) \leq \hat{C}$ requires the computation of a minimum-length picker tour serving $\hat{O}$. This part is the *Benders subproblem* and the violated tour constraint for $\hat{O}$ is a *Benders optimality cut*. Briant *et al.* solve the Benders subproblem with Steiner TSP models because the considered warehouses are more involved than in the standard benchmarks (they have a two-block parallel-aisle warehouse with the possibility that pickers walk diagonally).

Although the Benders approach looks very appealing, Briant *et al.* (2020, p. 502) report that 'the number of tour constraints generated [...] can grow considerably'. As a countermeasure, they add several tour constraints at a time and limit the number of Benders iterations. When the limit is reached without success (no negative reduced cost variable λ_w found), they strengthen $BM(\mathcal{S})$ using a *relaxation of the routing* which integrates a relaxed Steiner TSP model into $BM(\mathcal{S})$. Since solving the new Benders master is computationally very costly, Briant *et al.* allow only one iteration with the relaxation of the routing. Overall, the possibly incomplete pricing makes the algorithm of Briant *et al.* a heuristic for the JOBPRP. This is even more true, because column generation is not integrated into a branch-and-bound but the MIP solver computes an integer solution from the integer RMP.

4.1.2. Heuristic Pricing

Solving the pricing problem exactly in each column-generation iteration is computationally costly. Therefore, we complement exact pricing with the following three pricing heuristics.

First, we store the minimum lengths of all picker tours that have been computed in previous attempts to generate negative reduced-cost variables (in different pricing iterations) in a hash table. The hash key is a feasible batch $O' \subset O$, i.e., a subset of orders, fulfilling the capacity constraint $\sum_{o \in O'} q_o \leq Q$. The hash value stored for O' is the tour length denoted by $c(O')$. For convenience, we introduce the short-hand notation $\pi(O')$ for the sum $\sum_{o \in O'} \pi_o$, which is the profit that the batch generates. Then, the reduced cost $c_w(\boldsymbol{\pi})$ of the minimum-length picker tour serving O' can be computed quickly as $c(O') - \pi(O')$ for all entries of the hash table. If one or several order subsets have $c(O') - \pi(O') < 0$, the corresponding picker-tour variables are added to the RMP and pricing terminates on this first level of heuristic pricing.

Second, we invoke a simple metaheuristic similar to the approach proposed by Hintsch and Irnich (2020) for the SoftCluVRP. It exploits that the reduced cost $\tilde{c}_w(\boldsymbol{\pi})$ (see eq. (7)) is zero for all variables $\lambda_w, w \in \mathcal{W}$ that are basic variables (we assume that the RMPs are re-optimized with a variant of the simplex algorithm). The batch $O' = \{o \in O : \bar{z}_o = 1\}$ corresponding to $w = (\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}})$ is, therefore, a promising starting point when searching for similar batches with negative reduced cost. Algorithm 1 takes such a batch O' and tries to improve its reduced cost by adding and removing orders. Its heart is a *variable neighborhood descent* (VND, Mladenović and Hansen, 1997) that alternates between identifying a best order to add and a best order to drop from the current batch O' (Step 3 in Algorithm 1). Herein, a best order o is one that minimizes the reduced cost of $O'' = O' \cup \{o\}$ or $O'' = O' \setminus \{o\}$, respectively. The VND does not require $c(O'') - \pi(O'') < 0$. We look up the length $c(O'')$ in the hash table. If no entry exists, we compute the value $c(O'')$ with the DP of Ratliff and Rosenthal (saving the new value in the hash table afterward). Steps 6–7 randomly perturb the current batch by removing some orders (a random number between 1 and $n^{\text{VND}}_{\text{remove}}$ is drawn) so that the VND can be repeated for a given number $n^{\text{VND}}_{\text{iter}}$ of iterations. New variables are added to the RMP for all batches $O' \in \mathcal{S}$ returned by Algorithm 1. Only if the second level of the heuristic pricing fails, i.e., $\mathcal{S} = \varnothing$, the third level is invoked.

Algorithm 1: `VND-based Pricing Heuristic`(O')

Input: A feasible batch O', dual prices $\boldsymbol{\pi}$
Output: A set of negative reduced-cost batches $\mathcal{S}$

$\mathcal{S} \leftarrow \varnothing$
for $iter = 1, 2 \dots, n^{VND}_{iter}$ **do**
  VND(O')
  if $c(O') - \pi(O') < 0$ **then**
    $\mathcal{S} \leftarrow \mathcal{S} \cup \{O'\}$
  for *up to* n^{VND}_{remove} *times, as long as* $|O'| > 1$ **do**
    Randomly chose an order $o \in O'$ and set $O' \leftarrow O' \setminus \{o\}$
return $\mathcal{S}$

Third, the MIP solver is used with formulation (4) and a reduced state space (note that it is the *extended* state space described in Section 2.3 that is reduced). We observed that the aisle traversals `gap` and `2pass` are rarely part of a solution to the pricing problem, in particular for warehouses with long aisles. Therefore, we only consider the aisle traversals `top`, `bottom`, `1pass`, and `void`, if $C \geq n^{\text{redNet}}_{\text{cells}}$ (this is a simple check whether aisles are long). Likewise, only the cross-aisle traversals `00`, `11` are considered together with `20` (`02`) if the depot is located at the top (bottom). Additionally, the network is further reduced after some column-generation iterations: We count with $f(e)$ how often every edge traversal $e \in E$ is part of a pricing solution. The ratio $r(e) = f(e)/\sum_{e \in E} f(e)$ can be interpreted as the success rate of $e \in E$. As soon as the sum of traversals in pricing solutions exceeds a chosen threshold, i.e., $\sum_{e \in E} f(e) \geq n^{\text{redNet}}_{\text{thrshld}}$, we only consider traversals $e \in E$ with success rate $r(e) \geq \varepsilon^{\text{redNet}}$. Only if no negative reduced-cost variables can be identified with the reduced state space, the exact pricing algorithm of Section 4.1.1 is applied.

4.2. Cutting

The success of BPC algorithms in vehicle routing applications can be partly attributed to the invention of *subset-row inequalities* (SRIs, Jepsen *et al.*, 2008) because they often significantly strengthen the linear relaxation of the master program. SRIs apply to extensive formulations that have a set-packing substructure. For the JOBPRP, these are the batching constraints (6b) as already exploited in the work of Muter and Öncan (2015), who use SRIs for row subsets of size three only. In general, an SRI is defined by a subset $R = \{o_1, o_2, \dots, o_q\} \subseteq O$ of $q \geq 3$ different rows and weights $\mathbf{u} = (u_1, u_2, \dots, u_q)$ as

$$\sum_{w=(\bar{\mathbf{x}},\bar{\mathbf{y}},\bar{\mathbf{z}}) \in \mathcal{W}} \left\lfloor \sum_{j=1}^{q} \bar{z}_{o_j} u_{o_j} \right\rfloor \lambda_w \leq \left\lfloor \sum_{j=1}^{q} u_j \right\rfloor . \qquad \text{dual:} \quad [\tau_{(R,\mathbf{u})}] \tag{9}$$

Let $\tau_{(R,\mathbf{u})}$ be the dual of the SRI for $(R, \mathbf{u})$ when integrated into the master program (6). The presence of SRIs in the RMP impacts the reduced cost of a variable λ_w for $w = (\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}}) \in \mathcal{W}$ so that (7) becomes

$$\tilde{c}_w(\boldsymbol{\pi}) = \sum_{e \in E} c_e \bar{x}_e - \sum_{o \in O} \bar{z}_o \pi_o - \sum_{\substack{(R=\{o_1,o_2,\dots,o_q\},\\ \mathbf{u}=(u_1,u_2,\dots,u_q))}} \left\lfloor \sum_{j=1}^{q} \bar{z}_{o_j} u_{o_j} \right\rfloor \tau_{(R,\mathbf{u})} \tag{7'}$$

where the last sum is taken over all active SRIs defined by $(R, \mathbf{u})$.

Most of the works that use SRIs develop BPC algorithms in which the pricing problems are SPPRCs solved with labeling algorithms. In this context, Pecin *et al.* (2017a) provided an experimental analysis for the CVRP, and Pecin *et al.* (2017b) listed all combinations of non-dominated weights for $3 \leq |R| \leq 5$. Moreover, MIP solver-based approaches that enumerate columns with reduced cost not larger than the integrality gap use SRIs to tighten the lower bounds (see, e.g., Baldacci *et al.*, 2008; Muter and Öncan, 2015). The only BPC algorithm that uses SRIs and solves the pricing subproblem with a MIP solver is, as far as we know, the one of Hintsch *et al.* (2021). We briefly summarize their findings.

First, subsets R with more than four elements are generally not beneficial, since the computational effort for separation of violated SRIs is high, and the increase of the lower bound is often minor. Second, for $|R| = 3$ and $|R| = 4$, the controlled introduction of SRIs is superior to pure branch-and-price results. Third, the integration of SRIs into the pricing problem's MIP formulation is relatively simple, in particular if binary row-indicator variables are already present (like the z_o-variables in our subproblem (4)). The integration of the SRIs must modify the objective function to properly reflect the difference between (7) and (7'). Hintsch *et al.* presented two alternative modifications of the original pricing problem that fully guarantee correct reduced-cost computations and mutually do not dominate each other. Their computational experiments indicated that a combination of both modifications is also computationally advantageous. All this is in line with our pretest, and we present both modifications now.

For each active SRI defined by $(R, \mathbf{u})$, a non-negative integer variable $t_{R,\mathbf{u}}$ must be introduced. It models the coefficient of $\tau_{(R,\mathbf{u})}$ in the last sum of (7'). Hence, the objective (4a) must be extended by the extra term $\sum_{(R,\mathbf{u})} \tau_{R,\mathbf{u}} t_{R,\mathbf{u}}$ leading to

$$c(\boldsymbol{\pi}) = \min \sum_{e \in E} c_e x_e - \sum_{o \in O} \pi_o z_o + \sum_{(R,\mathbf{u})} \tau_{R,\mathbf{u}} t_{R,\mathbf{u}}. \tag{10a}$$

For $R = \{o_1, o_2, o_3\}$ and the unique undominated weights $(u_1, u_2, u_3) = (1/2, 1/2, 1/2)$, and likewise $R' = \{o_1, o_2, o_3, o_4\}$ and the unique undominated weights $(u'_1, u'_2, u'_3, u'_4) = (2/3, 1/3, 1/3, 1/3)$, the coupling between the z- and the t-variable can be accomplished via

$$\begin{array}{rl} z_{o_1} + z_{o_2} + z_{o_3} - 2t_{R,\mathbf{u}} \leq 1 \\ z_{o_1} + z_{o_2} \phantom{+ z_{o_3}} - t_{R,\mathbf{u}} \leq 1 \\ z_{o_1} \phantom{+ z_{o_2}} + z_{o_3} - t_{R,\mathbf{u}} \leq 1 \\ + z_{o_2} + z_{o_3} - t_{R,\mathbf{u}} \leq 1 \\ \forall (R, \mathbf{u}), |R| = 3 \end{array} \quad \text{and} \quad \begin{array}{rl} 2z_{o_1} + z_{o_2} + z_{o_3} + z_{o_4} - 3t_{R',\mathbf{u}'} \leq 2 \\ z_{o_1} + z_{o_2} \phantom{+ z_{o_3} + z_{o_4}} - t_{R',\mathbf{u}'} \leq 1 \\ z_{o_1} \phantom{+ z_{o_2}} z_{o_3} \phantom{+ z_{o_4}} - t_{R',\mathbf{u}'} \leq 1 \\ z_{o_1} \phantom{+ z_{o_2} + z_{o_3}} z_{o_4} - t_{R',\mathbf{u}'} \leq 1 \\ z_{o_2} + z_{o_3} + z_{o_4} - t_{R',\mathbf{u}'} \leq 1 \\ \forall (R', \mathbf{u}'), |R'| = 4 \end{array} \tag{10b}$$

where the first inequality (on the left and on the right) alone as well as the three/four last inequalities suffice. Their combination though gives an even stronger formulation. Overall, the SRI-adapted formulation replacing (4) comprises the new objective (10a), constraints (4b)–(4h), (10b), and $t_{R,\mathbf{u}} \in \{0, 1\}$ for all $(R, \mathbf{u})$.

We use a straightforward exact enumeration procedure to detect the most violated SRIs with $|R| = 3$. For subsets of size $|S| = 4$, we use the heuristic separation algorithm of Hintsch *et al.* (2021). Likewise, the general separation and cut selection strategy for violated SRIs is taken from the same work: The separation algorithms are invoked only up to levels $n^{\text{SRI}}_{\text{level:3}}$ and $n^{\text{SRI}}_{\text{level:4}}$ of the branch-and-bound tree for $|R| = 3$ and 4, respectively. Only SRIs violated by a minimum violation value ε^{SRI} are considered and not more than $n^{\text{SRI}}_{\text{round}}$ violated SRIs are added in one round of separation. We take the most violated SRIs and allow not more than $n^{\text{SRI}}_{\text{row}}$ SRIs that refer to the same order.

4.3. Branching

Let $(\hat{\lambda}_w)_{w \in \mathcal{W}}$ be a fractional solution of the RMP. We apply the following two-level branching scheme: At the first level, if the number $\hat{b} := \sum_{w \in \mathcal{W}} \hat{\lambda}_w$ of picker tours is fractional, we create two branches with $\sum_{w \in \mathcal{W}} \lambda_w \leq \lfloor \hat{b} \rfloor$ and $\sum_{w \in \mathcal{W}} \lambda_w \geq \lceil \hat{b} \rceil$. We noticed that for some JOBPRP instances of the benchmarks (see Section 5.3.1), the root node of the branch-and-bound produced a solution with $\hat{b} < \underline{b}$, i.e., less picker tours than provably required. Therefore, we initialize the RMP with the additional constraint $\sum_{w \in \mathcal{W}} \lambda_w \geq \underline{b}$ instead of possibly creating an infeasible child node later during branching.

At the second level, we apply Ryan-Foster branching (Ryan and Foster, 1981) on the set-partitioning constraints (6b). Gademann and van de Velde (2005) also used Ryan-Foster branching and described the implementation for the master and pricing problem: For each ordered pair of two different orders $o_1, o_2 \in O$, the value $\hat{t}_{o_1,o_2} := \sum_{w=(\mathbf{x},\mathbf{y},\mathbf{z}) \in \mathcal{W}: \bar{z}_{o_1} = \bar{z}_{o_2}} \hat{\lambda}_w$ indicates whether the orders are in different batches ($\hat{t}_{o_1,o_2} = 0$)

or in the same batch ($\hat{t}_{o_1,o_2} = 1$). Any value $0 < \hat{t}_{o_1,o_2} < 1$ can be used to branch on. In the together-branch, the two orders o_1 and o_2 can be merged into one new 'super order' (Gademann and van de Velde do this). The implementation is even simpler in our case because the constraint $z_{o_1} = z_{o_2}$ can be added to model (4) (or the SRI-adapted version, see the previous subsection). In the separate-branch, it must be ensured that the two orders o_1 and o_2 are not both selected. Gademann and van de Velde can cope with this requirement in their combinatorial branch-and-bound when adding a new order to a partial solution. We add the constraint $z_{o_1} + z_{o_2} \leq 1$ to model (4). In both branches, any incompatible variables λ_w must be removed from the RMP.

Many other works suggest to select one branching variable $\hat{t}_{o_1,o_2}$ closest to 0.5. We found that selecting a pair (o_1, o_2) of orders that maximizes $(1 - |\hat{t}_{o_1,o_2} - 0.5|) \cdot q_{o_1} \cdot q_{o_2}$ often leads to smaller branch-and-bound trees. The idea is that orders with a large weight have a strong impact on the linear relaxation. Moreover, a best-bound first tree exploration strategy is applied to improve the dual bound as fast as possible.

4.4. MIP Solver Heuristic

The best-bound first tree exploration strategy (see above) lets the BPC algorithm tend to identify integer solutions late, i.e., closely before regularly terminating. Hence, we also solve the RMP as an integer program with the MIP solver. To limit the computational effort, the MIP solver is invoked only for branch-and-bound nodes up to level $n^{\text{MIP}}_{\text{level}}$ (the root node is level 1), and a computation time limit of t^{MIP} seconds is imposed per node. The best computed integer solution (if any) is compared against the UB, which is updated if a new best solution has be returned by the MIP solver.

5. Computational Results

In this section, we first specify details of the computational setup and implementation. Afterwards, results for the SPRP-SS (Section 5.2) and the JOBPRP (Section 5.3) are analyzed and discussed.

5.1. Details of the Implementation

All algorithms are implemented in `C++` using the callable library of `CPLEX` 20.1.0 and compiled into 64-bit single-thread release code with Microsoft Visual Studio 2015. The computational study is performed on a 64-bit Microsoft Windows 10 computer with an Intel® Core™ i7-5930k CPU clocked at 3.5 GHz and 64 GB of RAM. In all calls to the MIP solver, `CPLEX`'s default values of all parameters are kept except for the time limit and setting the number of available threads to one. In the BPC algorithm for the JOBPRP, the RMPs are re-optimized with the primal simplex algorithm of `CPLEX`. For pricing, all integer solutions with negative reduced cost found by `CPLEX` are saved and the corresponding columns are added to the RMP. `CPLEX` is also used as a primal MIP-based heuristic for the JOBPRP in some branch-and-bound nodes (see Section 4.4) with a time limit of 60 seconds using all columns generated up to this point.

5.2. Results for Optimal Picker Routing and Scattered Storage

We first present the SPRP-SS benchmark instances and subsequently the computational results.

5.2.1. Benchmark Instances

The recent articles of Weidinger (2018) and Goeke and Schneider (2021) describe how different instances of the SPRP-SS can be generated (we have summarized the procedure in Section Appendix A of the Appendix). The layout is that of a parallel-aisle single-block warehouse with distance 3 between neighboring aisles and distance 1 between neighboring positions. An instance is characterized by a combination (m, C, n, α) of the four values number m of aisles, number C of cells per aisle, number n of different articles to be collected, and the scatter factor α. The scatter factor α indicates how often on average identical articles occur at different pick positions. Note that $\alpha = 1$ is the case of the classical SPRP without scattered storage.

Due to the ABC class-based instance generation process (see Section Appendix A), a pick list tends to have more order lines with A-articles than with B- and C-articles. Moreover, A-articles tend to have more pick positions compared to B- and C-articles. As a result, a typical pick list with n order lines has more than αn corresponding pick positions (the reader might have expected exactly αn positions). For example, instances with $\alpha = 10$ and $n = 30$ have on average 656.9 pick positions.

With the above procedure, Goeke and Schneider (2021) generated SPRP-SS instances with the goal to perform a kind of stress test for their new formulation. The largest instances had $m = 100$ aisles (in practice, this huge number of aisles is probably not served together with a single picker). Moreover, the largest scatter factor was chosen as $\alpha = 50$. For a (large) warehouse with $m = 25$ aisles, the result is that each article occurs on average twice in each aisle.

We decided to generate instances closer to the real-world application with a different set of combinations (m, C, n, α):

- $m \in \{5, 10, 25, 50\}$: we test 10 and 50 instead of 100;
- $C \in \{30, 60, 180\}$: as in (Goeke and Schneider, 2021);
- $n \in \{3, 7, 15, 30\}$: as in (Goeke and Schneider, 2021);
- $\alpha \in \{1, 2, 5, 10\}$: we restrict the combinations by requiring $\alpha < m$. This restriction makes the instances more realistic, since otherwise a typical batch can just be collected from one or two aisles close to the I/O point.

In total, this gives 156 feasible combinations (m, C, n, α). To obtain statistically firm results, we generated 50 instances per combination. Overall, this benchmark set comprises $156 \cdot 50 \cdot 2 = 15600$ instances (the factor two is for unit and varying demand) available at `https://logistik.bwl.uni-mainz.de/research/benchmarks/`.

5.2.2. Computational Results

The only competitive exact algorithm for the SPRP-SS is, up to date, the MIP solver-based solution of the model presented by Goeke and Schneider (2021). This approach (denoted by GS the sake of brevity) is now compared with the MIP solver-based solution of our network-flow (NF) formulation (3). We implemented and tested the two formulations, making sure both GS and NF find identical optimal objective values for all instances.

We start with the experimental comparison of one special case. For the scatter factor $\alpha = 1$, the resulting problem reduces to a basic (un-scattered) SPRP, and the subcases of sufficiently many items and varying supply and varying demand do not need to be distiguished, since they lead to identical instances (recall that we replaced b_{se} by $\min\{d_s, b_{se}\}$ for all $s \in S$ and $e \in E$, see Section 2.2). Moreover, in this special case, the original DP algorithm of Ratliff and Rosenthal constitutes a third solution approach. For each group of instances with identical parameters (m, C, a), let t_{DP} denote the average computation time (in milliseconds) for the solution of the respective DP and likewise t_{NF} for NF. Table 2 displays t_{DP}, t_{NF}, and $t_{\text{GS}}/t_{\text{NF}}$. The latter value is the geometric mean of the ratios of GS and NF solution times (over the 50 instances of each group). It is an estimate for the speedup one can expect when replacing GS by NF. In particular, all values $t_{\text{GS}}/t_{\text{NF}} > 1$ indicate that NF is faster than GS.

It is not surprising that DP is always the fastest algorithm, where all average computation times stay below 2.5 milliseconds. Moreover, the speedup values $t_{\text{GS}}/t_{\text{NF}} > 1$ are between 5.5 and 24.1, with the trend that speedups are greater of larger warehouses (more aisles and more cells per aisle), and the reverse trend can be observed for longer pick lists (increasing a-values). The general superiority of NF over GS can be explained by the fact that the GS model does not have the integrality property, while NF as a pure network-flow model is having it. As a result, the MIP solver needs to not only solve the linear relaxation of the GS model, but it must close the integrality gap by cutting and branching, which takes more time than the solution of the LP providing a solution of NF directly.

For the true scattered case ($\alpha > 1$), Table 3 and Table B.8 in the Appendix show aggregated results for sufficiently many SKUs as well as varying supply and varying demand, respectively. The comparison is only between GS and NF, since the direct DP approach is no longer applicable. The overall average speedup of NF over GS is 3.6. For both NF and GS, the trend is that instances with a higher number n of SKUs in the pick list, a larger warehouse (more aisles and cells), and a larger scatter factor α are more difficult to solve optimally, i.e., their solution by the MIP solver consumes more time. One can however expect computation

Table 2: Comparison of computation times (in milliseconds) of DP and NF as well as speedup factor comparing GS and NF for the basic SPRP (non-scattered).

Warehouse dimension	Number of SKUs in pick list											
	$a = 3$			$a = 7$			$a = 15$			$a = 30$		
(m, C)	t_{DP}	t_{NF}	$t_{\text{GS}}/t_{\text{NF}}$	t_{DP}	t_{NF}	$t_{\text{GS}}/t_{\text{NF}}$	t_{DP}	t_{NF}	$t_{\text{GS}}/t_{\text{NF}}$	t_{DP}	t_{NF}	$t_{\text{GS}}/t_{\text{NF}}$
$(5, 30)$	0.3	0.8	8.5	0.3	1.0	6.2	0.3	1.1	6.1	0.3	1.2	5.6
$(5, 60)$	0.3	0.8	8.5	0.3	1.0	7.5	0.4	1.2	5.5	0.4	1.2	5.6
$(5, 180)$	0.3	0.9	8.9	0.3	1.0	6.7	0.3	1.1	5.9	0.2	1.1	5.9
$(10, 30)$	0.4	1.4	9.0	0.4	1.9	8.2	0.5	2.2	6.6	0.6	2.3	7.0
$(10, 60)$	0.4	1.4	9.3	0.4	1.8	7.1	0.5	2.3	6.2	0.5	2.5	7.3
$(10, 180)$	0.4	1.3	8.4	0.5	1.8	9.8	0.4	2.4	7.0	0.5	2.4	6.9
$(25, 30)$	0.8	3.6	14.6	0.9	4.5	10.4	1.0	6.0	7.3	1.2	6.7	7.2
$(25, 60)$	1.1	3.6	21.7	1.0	4.3	15.2	1.0	5.3	9.5	0.9	7.2	8.1
$(25, 180)$	0.8	3.3	17.5	0.8	4.1	16.2	1.2	5.3	10.5	1.4	6.7	8.8
$(50, 30)$	2.1	7.3	14.4	1.7	8.6	12.8	1.9	11.1	11.8	2.0	13.5	7.1
$(50, 60)$	1.8	8.5	22.5	1.7	8.9	23.2	2.0	10.6	14.4	2.5	12.8	10.4
$(50, 180)$	1.8	7.8	20.4	1.8	8.3	24.1	2.0	10.0	16.6	1.9	12.2	20.1

times below one second for (realistically sized) warehouses with not more than 25 aisles. Even the most difficult setting for $(m, C, a, \alpha) = (50, 180, 30, 10)$ has average computation times below 10 seconds.

Table 3: Comparison of GS and NF with computation times (in milliseconds) of GS and speedup factor comparing GS and NF for the SPRP-SS and sufficiently many items (unit demand; $b_{se} = d_s = 1$).

Scatter factor	Warehouse dimension	Number of SKUs in pick list							
		$a = 3$		$a = 7$		$a = 15$		$a = 30$	
	(m, C)	t_{NF}	$t_{\text{GS}}/t_{\text{NF}}$	t_{NF}	$t_{\text{GS}}/t_{\text{NF}}$	t_{NF}	$t_{\text{GS}}/t_{\text{NF}}$	t_{NF}	$t_{\text{GS}}/t_{\text{NF}}$
$\alpha = 2$	$(5, 30)$	5.5	3.7	7.2	2.8	14.3	1.2	23.5	1.0
	$(5, 60)$	5.1	4.6	8.2	3.2	13.7	1.7	29.9	1.0
	$(5, 180)$	5.0	4.7	8.5	5.0	16.7	1.8	36.6	0.7
	$(10, 30)$	9.4	5.3	14.8	4.3	22.5	2.4	41.2	1.1
	$(10, 60)$	9.2	7.3	16.6	5.3	23.1	2.8	44.2	1.3
	$(10, 180)$	8.6	8.8	14.1	7.8	22.0	2.6	49.2	1.1
	$(25, 30)$	16.5	6.9	23.4	6.0	31.3	3.8	65.7	2.4
	$(25, 60)$	15.2	10.4	22.4	7.1	26.8	4.5	47.6	2.7
	$(25, 180)$	16.5	11.0	23.4	13.0	33.3	6.8	52.6	3.3
	$(50, 30)$	24.1	9.6	28.1	10.2	39.2	5.1	53.7	3.9
	$(50, 60)$	24.2	11.9	29.8	12.4	38.8	9.7	49.8	4.5
	$(50, 180)$	31.8	35.3	37.3	39.8	34.8	27.6	58.7	8.4
$\alpha = 5$	$(10, 30)$	13.9	4.5	43.9	3.8	89.8	1.7	154.3	0.8
	$(10, 60)$	18.4	5.4	44.2	4.6	124.5	2.6	274.9	0.9
	$(10, 180)$	15.1	5.7	37.7	5.0	131.3	3.2	419.8	1.0
	$(25, 30)$	39.3	5.7	109.4	3.8	174.2	1.9	483.9	0.9
	$(25, 60)$	44.2	4.8	138.8	4.6	228.2	3.4	623.9	1.1
	$(25, 180)$	37.7	10.3	144.9	10.9	296.2	7.7	644.5	1.9
	$(50, 30)$	74.1	3.8	142.1	4.5	206.5	2.8	456.2	2.1
	$(50, 60)$	85.6	8.6	153.2	12.0	252.3	5.4	634.8	3.6
	$(50, 180)$	67.0	10.2	165.5	13.9	404.9	14.4	645.7	5.2
$\alpha = 10$	$(25, 30)$	184.8	2.9	648.5	1.5	1807.9	1.1	3008.6	0.8
	$(25, 60)$	181.2	3.8	1104.9	1.8	3413.3	1.1	4925.3	0.9
	$(25, 180)$	150.8	5.0	813.2	3.4	3388.7	3.1	6726.6	2.3
	$(50, 30)$	525.5	1.7	2424.6	2.4	2331.4	1.7	4989.8	1.0
	$(50, 60)$	660.5	2.5	2076.2	3.7	3530.8	2.2	6173.1	1.4
	$(50, 180)$	516.6	3.4	1465.9	8.2	4447.5	4.5	8299.9	4.4

Looking more closely at the case of sufficiently many items (unit demand, Table 3), NF is superior to GS ($t_{\text{NF}} < t_{\text{GS}}$) for almost all groups. Some exceptions occur for pick lists of size $n = 30$ and warehouses with

$m \leq 25$ aisles. Even in these cases, the acceleration factor $t_{\text{GS}}/t_{\text{NF}}$ is always greater or equal to 0.7. For the example of $(m, C, a, \alpha) = (5, 180, 30, 2)$ with $t_{\text{GS}}/t_{\text{NF}} = 0.7$, the 30 articles are stored at (on average) $112.1 > 60 = 2 \cdot 30$ positions in a narrow warehouse with only five aisles but 180 cells per aisle.This is a rather obscure setting. On the other extreme, the highest average speedup $t_{\text{GS}}/t_{\text{NF}} = 39.8$ was obtained for instances for $(m, C, n, \alpha) = (50, 180, 7, 2)$. Indeed, the speedup mainly depends on the warehouse size: the greater m and C, the greater $t_{\text{GS}}/t_{\text{NF}}$.

For instances with varying demand (Table B.8), the trends for what makes instances difficult is the same as in the unit-demand case. However, computation times t_{NF} are longer, e.g., the smallest (largest) times of 5 milliseconds (8.3 seconds) for the unit-demand case rise to 47 milliseconds (20.6 seconds). On average, varying demand leads to a 4.7 and 2.8 times slower solution of NF and GS, respectively. The factor $t_{\text{GS}}/t_{\text{NF}}$ shows similar trends as for unit demand, but the absolute speedup is lower. The highest value amounts to $t_{\text{GS}}/t_{\text{NF}} = 16.1$, the lowest to 0.5. The 19 cases (of 108) where NF is inferior to GS, i.e., $t_{\text{GS}}/t_{\text{NF}} < 1$, the pick list is relatively long (three cases for $a = 15$ and 16 cases for $a = 30$).

Finally, we analyze to which extend the computation time vary. To this end, we compute the *coefficient of variation* (CT) of the computation times for each group (50 instances per parameter combination (m, C, n, α)). Over the 156 groups, the minimum, average, and maximal CT-values are 0.33, 0.78, and 2.20 for GS compared to 0.10, 0.65, and 1.86 for NF, respectively. Overall, the computation times of NF are more predictable and stable than for GS.

In total, NF outperforms GS with the exception of a few extreme parameter combinations mentioned above.

5.3. Results for the Joint Order Batching and Picker Routing Problem

We first discuss the benchmark sets for the JOBPRP and explain how pretests were conducted. Afterwards detailed computational results are presented for each benchmark set.

5.3.1. Benchmark Instances and Pretests

Table 4 provides an overview of the characteristics of the three JOBPRP benchmark sets of (i) Bahçeci and Öncan (2021), (ii) Henn *et al.* (2010); Henn and Wäscher (2012) (in the following referred to as Henn *et al.*), and (iii) Muter and Öncan (2015). We explore the differences and effects of the instance characteristics on the BPC performance in more detail in separate computational results sections.

Table 4: Characteristics of the JOBPRP benchmark sets. All instances assume unit weights for SKUs.

	Bahçeci and Öncan			Henn *et al.*			Muter and Öncan		
Number of aisles m	10			10			10		
Number of cells C per aisle	10			45			10		
Picker capacity Q	{5,7,10,14,20,28,30}			{30,45,60,75}			{24,36,48}		
Number of orders $\lvert O\rvert$	{8,12}			{20,30,40,...,100}			{20,30,40,...,100}		
	min	avg	max	min	avg	max	min	avg	max
Minimal number of batches $\underline{b}$	1	2.7	6	3	19.1	59	3	11.4	26
Number of SKUs a	10	18.8	29	125	310.6	442	62	93.0	100
Number of order lines l	20	26.0	30	212	855.4	1595	102	362.2	623
Smallest order size	1	2.2	3	4	5.1	10	2	2.0	2
Average order size	2.5	2.8	3.5	10.6	14.3	17.8	5.1	6.1	7.0
Largest order size	3	3.3	4	20	23.7	24	10	10.0	10

Bahçeci and Öncan (2021) propose a benchmark set of 1350 instances. Groups of instances represent five different storage location policies, four of which are ABC class-based. The warehouse size is identical for all instances with $m = 10$ aisles and $C = 10$ cells per aisle. We refer to (Bahçeci and Öncan, 2021) for a detailed description of how the instances were generated.

The second benchmark set is proposed by Henn *et al.* (2010); Henn and Wäscher (2012) with a warehouse of $m = 10$ aisles and $C = 45$ cells per aisle yielding $2 \cdot 10 \cdot 45 = 900$ pick positions. Also here, two different storage location assignments are considered, i.e., *uniformly distributed SKUs* (UDS) and *class-based distributed SKUs* (CBDS). For CBDS, SKUs with a high turnover rate are stored closer to the I/O point (located at the leftmost aisle). A-articles are stored in the depot aisle, B-articles right next in aisles 2 to 4, and C-articles in aisles 5 to 10. The classification assigns 10%, 30%, and 60% of the articles to classes A, B, and C, respectively, and they account for 52%, 36%, and 12% of the demand. The number of SKUs per order is randomly drawn from the interval $[5, 25]$. Henn *et al.* distinguish between the routing policies S-shape (=traversal) and largest gap and create instances for both policies. However, the instances themselves can be solved with any policy (this is what we do in the following). With nine different numbers of orders (between 20 and 100), four picker capacities (between 30 and 75, see Table 4), and the two different storage location assignments, the benchmark consists of 72 groups of instances generated with identical parameter combination. The original benchmark set consists of 80 instances for each combination, resulting in a (huge) benchmark set of 5760 instances. In order to limit the computational effort (we want to grant one hour of computation time per instance), we consider a proper subset by solving only the 20 first instances for each combination/group. Hence, we solve 1440 instances in total.

The third benchmark set Muter and Öncan (2015) also assumes a parallel-aisle single-block warehouse with $m = 10$ aisles and $C = 10$ cells per aisle, yielding $10 \cdot 10 = 100$ pick locations (one article is stored in each of the $m \cdot C$ possible pick positions). The variation of the picker capacities $Q \in \{24, 36, 48\}$ leads to increasingly difficult instances. The second parameter used to form groups of instances is the number $|O|$ of orders (between 20 and 100 in steps of 10). The SKUs are uniformly distributed in the warehouse, and the number a of SKUs per order is randomly drawn from the interval $[2, 10]$. With 10 instances per group, the benchmark comprises 270 instances in total.

For the sake of clarity and reproducibility of our results as well as to support future research, we have converted the original instance files into a uniform and clean format. All instances are online available at https://logistik.bwl.uni-mainz.de/research/benchmarks/.

Pretests. For the BPC algorithm, we performed pretests with a subset of 30 instances (of the three benchmarks) to identify reasonable parameter values. Starting with a default setting, we only change the value of one parameter while keeping the remaining parameters fixed. The procedure is repeated for each parameter after selecting the value with the best average computation time in each case. The parameter values we obtained are given in Table 5.

Table 5: Parameters of the BPC algorithm for the JOBPRP.

Algorithmic Component											
Pricing, VND		Pricing, red. Network			SRIs					MIP solver	
$n^{\text{VND}}_{\text{iter}} = 100$	$n^{\text{VND}}_{\text{remove}} = 3$	$n^{\text{redNet}}_{\text{cells}} = 30$	$n^{\text{redNet}}_{\text{thrshld}} = 1000$	$\varepsilon^{\text{redNet}} = 0.001$	$\varepsilon^{\text{SRI}} = 0.25$	$n^{\text{SRI}}_{\text{level:3}} = 5$	$n^{\text{SRI}}_{\text{level:4}} = 2$	$n^{\text{SRI}}_{\text{round}} = 10$	$n^{\text{SRI}}_{\text{row}} = 5$	$n^{\text{MIP}}_{\text{level}} = 15$	t^{MIP} [s] $= 60$

5.3.2. Computational Results for the Benchmark Instances of Bahçeci and Öncan

Bahçeci and Öncan (2021) created a benchmark set to test their MIP formulations for the JOBPRP with the policies traversal, return, midpoint, largest gap, composite, mixed, and optimal. We implemented all policies except for the policy mixed.

For our BPC algorithm, the policy optimal turned out to be the most computationally costly. However, all 1350 instances can be solved to proven optimality within 40 seconds, and the average computation time is less than one second. The instances can therefore be considered as easy to solve. Table 4 explains this well, because the maximal number of orders is $|O| = 12$, the maximal number of SKUs is $a = 29$, and the minimal number of batches is $\underline{b} = 6$, which is small in comparison to the other benchmark sets. Bahçeci and Öncan report average computation times of 912 seconds for their MIP formulation. For the sake of brevity, we refrain from reporting detailed results which are however online available at https://logistik.bwl.uni-mainz.de/research/benchmarks/.

The BPC computation times for the heuristic policies are on average smaller, which can be explained by the smaller underlying state space used in model (4) (see Korbacher *et al.*, 2022).

5.3.3. Computational Results for the Benchmark Instances of Henn et al.

We now consider the benchmark set introduced and studied first in (Henn *et al.*, 2010; Henn and Wäscher, 2012). In the literature, the instances are solved heuristically with the policies traversal and largest gap only. We therefore focus also on these two policies and compare the results to those that we obtain with the policy optimal.

Aggregated results for storage location assignment UDS are presented in Table 6 and for CBDS in Table C.9 of the Appendix, while instance-based results for both storage location assignments are reported in the Appendix in Table C.10. All aggregated results are grouped by the picker capacity Q and the number $|O|$ of orders, and the table entries have the following meaning:

- **#inst:** number of instances;
- **#opt:** number of instances solved to proven optimality within 1 hour (3600 seconds);
- **time $\bar{t}$:** average computation time in seconds (unsolved instances are taken into account with the time limit TL of 3600 seconds);
- **gap:** the average remaining gap $100 \cdot (UB - LB)/LB$ in percent at termination (the average is taken only over the instances for which a lower bound LB and upper bound UB was computed with the BPC algorithm);
- **$\Delta_{\textbf{Pol}}$:** the average relative difference (in percent) between the optimal objective value z_P of the respective heuristic routing policy P and the optimal objective value z^* of the policy optimal, computed as $100 \cdot (z_P - z^*)/z^*$ (only instances solved to proven optimality for both policies are taken into account).

For policies traversal and largest gap, some of the instances were solved with the ALNS/TS metaheuristic of Žulj *et al.* (2018), for which Table C.10 shows the objective function value.

Results for Instances with uniformly distributed SKUs. We start with a discussion of the results for storage location assignment UDS (see Table 6). The performance of the BPC algorithm mainly depends on the capacity-to-demand ratio, i.e., instances with a larger capacity Q are more difficult to solve. Likewise, but secondary, instances with a higher number of orders are more difficult. For the policy optimal, the BPC algorithm solves 177 of the 720 instances to proven optimality with an average computation time of 2898.9 seconds. In comparison with the policies traversal and largest gap, more instances (281 and 198) are solved exactly, and the average computation times are smaller (2315.2 and 2845.8 seconds, respectively). This can again be explained with the relatively smaller extended state spaces of the heuristic policies, over which the respective formulation (4) can be solved faster. Note that pricing contributes to more than 95% of the overall BPC computation time. Moreover, the extended state space for policy traversal is typically smaller than the one for policy largest gap: The warehouse characteristic is $m = 10$ aisles and $C = 45$ cells per aisle, so that there exist many edge traversals of type gap for the policy largest gap which are irrelevant for traversal. This is consistent with the analysis conducted in Section 2.3 which has shown that edge traversals of type gap are the main driver of the size of the extended state spaces and model (4).

The routing policies also have an influence on the (average) remaining gap of the BPC algorithm at termination. For traversal, the average remaining gap of 0.9% is very small and highlights the good performance of the BPC algorithm for this policy. The remaining gap of instances with policy largest gap is significantly larger (53.7%), also compared to policy optimal (19.7%). Numbers should however be considered with care, because different instances are solved optimally with the different policies and upper bounds are differently tight for the three policies.

Previous works could only rely on solutions of small-sized JOBPRP instances for a precise total cost comparison. With the results we obtained for several medium-sized and even larger instances, it is now for

Table 6: Results for the benchmark set of Henn *et al.* with storage location assignment UDS.

Instances			Policy										
			optimal			traversal				largest gap			
Q	$\|O\|$	#inst	#opt	time $\bar{t}$	gap	#opt	time $\bar{t}$	gap	Δ_{Pol}	#opt	time $\bar{t}$	gap	Δ_{Pol}
30	20	20	20	46.7	0.0	20	10.1	0.0	17.9	20	7.1	0.0	8.8
	30	20	20	104.1	0.0	20	25.2	0.0	18.3	20	19.1	0.0	8.9
	40	20	19	408.2	<0.1	20	9.3	0.0	17.6	20	388.7	0.0	8.8
	50	20	18	837.0	<0.1	19	281.7	<0.1	17.7	19	1168.4	<0.1	8.7
	60	20	19	1213.5	<0.1	18	387.3	<0.1	17.6	17	1650.1	<0.1	8.8
	70	20	14	1579.9	0.3	17	612.7	<0.1	18.4	13	1876.6	0.2	8.9
	80	20	9	2663.8	2.0	16	828.3	<0.1	17.6	10	2820.4	0.3	9.0
	90	20	10	2464.7	1.0	15	926.9	<0.1	16.7	4	3449.8	1.7	9.0
	100	20	9	2529.8	1.8	17	613.2	<0.1	17.7	6	3199.4	1.5	9.0
45	20	20	15	1509.7	0.2	19	366.9	<0.1	10.1	20	273.0	0.0	12.7
	30	20	8	2833.8	0.4	16	1182.5	0.1	10.2	15	1619.4	<0.1	12.8
	40	20	1	3513.1	1.1	12	1618.2	0.2	8.6	3	3392.4	0.5	
	50	20	0	*TL*	6.4	10	2341.2	0.2		0	*TL*	3.6	
	60	20	0	*TL*	15.7	13	2103.8	0.1		0	*TL*	12.3	
	70	20	0	*TL*	16.5	6	2776.8	0.2		0	*TL*	18.9	
	80	20	0	*TL*	20.3	4	3121.0	0.1		0	*TL*	28.3	
	90	20	0	*TL*	22.8	2	3388.4	0.2		0	*TL*	27.8	
	100	20	0	*TL*	21.5	0	*TL*	0.1		0	*TL*	33.3	
60	20	20	5	2934.0	1.2	8	2272.9	1.2	8.0	14	1831.2	0.2	15.8
	30	20	2	3439.9	1.9	7	2579.9	1.0	8.4	5	2991.1	0.9	17.4
	40	20	0	*TL*	7.7	4	2966.1	1.1		0	*TL*	5.4	
	50	20	0	*TL*	21.2	4	3049.9	0.6		0	*TL*	23.0	
	60	20	0	*TL*	33.1	1	3496.2	0.8		0	*TL*	51.1	
	70	20	0	*TL*	38.4	1	3534.4	0.7		0	*TL*	53.8	
	80	20	0	*TL*	32.1	0	*TL*	0.6		0	*TL*	79.8	
	90	20	0	*TL*	40.2	0	*TL*	1.4		0	*TL*	114.0	
	100	20	0	*TL*	41.6	0	*TL*	1.6		0	*TL*	194.8	
75	20	20	6	2741.2	2.0	5	2736.8	3.7	5.3	12	2160.5	0.7	21.2
	30	20	2	3542.3	4.0	5	2757.5	2.4	5.5	0	*TL*	2.3	
	40	20	0	*TL*	12.5	2	3359.2	2.4		0	*TL*	14.2	
	50	20	0	*TL*	31.6	0	*TL*	1.6		0	*TL*	54.3	
	60	20	0	*TL*	56.2	0	*TL*	1.1		0	*TL*	98.0	
	70	20	0	*TL*	64.4	0	*TL*	2.3		0	*TL*	241.9	
	80	20	0	*TL*	73.6	0	*TL*	3.9		0	*TL*	327.5	
	90	20	0	*TL*	74.5	0	*TL*	2.9		0	*TL*	418.4	
	100	20	0	*TL*	80.3	0	*TL*	3.6		0	*TL*	386.2	
Total		720	177	2898.9	19.7	281	2315.2	0.9	15.9	198	2845.8	53.7	10.2

the first time possible to compare policy optimal with optimal solutions for heuristic routing policies in the order batching context. For example, for a picker capacity of $Q = 30$, the picker tours of the policy traversal (largest gap) are approximately 17% (9%) longer compared to the optimal routing. For higher capacities $Q \geq 60$ and accordingly longer picker tours, the total cost of traversal is smaller than the total cost of largest gap. This can be explained with on average longer picker tours in which more SKUs have to be picked per aisle such that traversing an aisle is more efficient than entering an aisle from both ends. The brief cost comparison illustrates that the choice of a good policy depends not only on storage assignment and warehouse geometry but also on the picker capacity. Note that for these unit-demand instances, the residual capacity almost always fulfills $Q - n \leq 5$ where n is the length of a pick list of a batch in the solution. Taking this fact into account, our results are in line with those of Petersen (1997), who assumes a warehouse with geometry $m : C = 1 : 2$ and pick lists of length $n \leq 45$ (or accordingly a picker capacity $Q \leq 45$).

Results for Instances with class-based distributed SKUs. Table C.9 in the Appendix reports results for instances with storage location assignment CBDS. Overall, these JOBPRP instances are less difficult to solve with our BPC algorithm with the same trend that the primary driver of the practical difficulty is the capacity Q and the secondary driver the number of orders. For the sake of brevity, we only report some key figures: For the policies optimal, traversal, and largest gap, 244, 359, and, 315 instances can be solved to proven optimality with an average computation time of 2573.1, 1982.7, and 2239.6 seconds, respectively. Hence, the BPC algorithm performs best for the policy traversal. The average values of the remaining gap are 9.9%, 0.8%, and 6.1% for the policies optimal, traversal, and largest gap, respectively. Especially for $Q = 30$ the values are very low with gap≤ 0.1.

For for all routing policies and CBDS, total costs are on average lower than for UDSas high-demanded articles are located close to the depot. In contrast to UDS, the average relative difference Δ_{Pol} to optimal routing is on average higher (lower) for traversal (largest gap). Policy largest gap is advantageous for $Q \leq 60$ and traversal for $Q = 75$.

Comparison with the ALNS/TS metaheuristic of Žulj et al.. The ALNS/TS metaheuristic of Žulj *et al.* (2018) handles two routing policies traversal and largest gap. For these, the BPC algorithm finds new best known solutions for 185 and 101 (of 320) instances, respectively. Moreover, we evaluated the cases where the upper bound UB_{BPC} provided by the BCP algorithm at termination is worse than the solution $UB_{\text{ALNS/TS}}$ of the metaheuristic. The average relative difference $(UB_{\text{BPC}} - UB_{\text{ALNS/TS}})/UB_{\text{ALNS/TS}}$ is 1.8% and 2.8% for the two routing policies, respectively. Given that the tree-search strategy of the BPC algorithm focuses on lower-bound improvement (see Section 4.3), these differences can be considered rather small.

5.3.4. Computational Results for the Benchmark Instances of Muter and Öncan

Results for the benchmark set of Muter and Öncan (2015) are summarized in Table 7. Our study includes all policies, while Muter and Öncan studied only the policies traversal, midpoint, and return. The instances of Muter and Öncan tend to be more difficult than the instances of Henn *et al.* because the average number of orders per batch is higher so that the batching decision becomes more complicated (see also Table 4). In total, we compute 246 provably optimal solutions compared to 201 computed by Muter and Öncan. Note that Muter and Öncan reported only aggregated results for groups of instance with identical capacity Q and number $|O|$ of orders. No detailed results per instance were published. However, it is interesting to perform a groupwise comparison. To this end, Table D.11 in the Appendix lists the number of optima per group. Only for the policies midpoint and return and the smallest capacity $Q = 24$, the algorithm of Muter and Öncan is able to compute 3 and 6 more optimal solutions than the BPC algorithm. We have two possible explanations: First, the combinatorial branch-and-bound algorithm that they use for pricing can better handle small capacities. Second, also the column-enumeration part of their approach benefits from smaller capacities, because less feasible batches exist. Accordingly, Henn and Wäscher (2012) mention that with a brute-force enumeration of all feasible batches they were able to solve some JOBPRP instances with up to 40 customer orders using the set-partitioning formulation. Table D.11 shows that the BPC algorithm outperforms the algorithm of Muter and Öncan for larger capacities $Q \in \{36, 48\}$.

Table 7: Results for the benchmark set of Muter and Öncan (MÖ). Note that the routing policy **composite** is defined as in (Korbacher *et al.*, 2022).

			Policy																		
			optimal			traversal				largest gap				midpoint				return				composite			
Q	$\|O\|$	#inst	#opt	time $\bar{t}$	gap	#opt	time $\bar{t}$	gap	Δ_{Pol}	#opt	time $\bar{t}$	gap	Δ_{Pol}	#opt	time $\bar{t}$	gap	Δ_{Pol}	#opt	time $\bar{t}$	gap	Δ_{Pol}	#opt	time $\bar{t}$	gap	Δ_{Pol}
24	20	10	10	408.9	0.0	10	210.2	0.0	13.5	10	131.7	0.0	5.2	10	49.7	0.0	8.5	10	58.3	0.0	36.3	10	137.9	0.0	0.9
	30	10	4	2613.5	0.6	8	1302.9	0.2	12.2	8	1476.2	0.1	5.6	9	801.9	<0.1	9.6	9	762.1	<0.1	37.8	6	2106.1	0.4	0.7
	40	10	6	2497.5	0.3	8	1038.4	0.1	12.6	8	1172.3	0.1	5.4	9	787.2	<0.1	9.3	9	611.2	<0.1	34.0	7	1513.9	0.1	0.6
	50	10	0	*TL*	0.6	5	2334.5	0.2		6	2383.7	0.1		8	1364.2	0.1		6	2033.6	0.2		2	3228.3	0.3	
	60	10	1	3449.9	0.5	8	1421.4	0.3	10.6	6	2298.6	0.2	5.5	6	2067.0	0.1	10.2	8	1749.1	0.2	35.1	6	2470.0	0.3	0.4
	70	10	0	*TL*	1.3	5	2614.2	0.4		2	3273.5	0.4		4	3015.8	0.3		4	3073.9	0.3		1	3447.8	0.6	
	80	10	0	*TL*	2.0	3	2983.8	0.2		1	3494.0	0.3		1	3407.9	0.2		3	3412.3	0.3		0	*TL*	0.8	
	90	10	0	*TL*	2.2	2	3441.6	0.4		0	*TL*	0.5		0	*TL*	0.6		1	3548.8	0.5		0	*TL*	1.0	
	100	10	0	*TL*	4.0	0	*TL*	0.7		0	*TL*	0.6		0	*TL*	1.0		0	*TL*	0.5		0	*TL*	2.1	
Subtotal		90	21	2996.6	1.3	49	2105.2	0.3	12.9	41	2381.1	0.2	5.4	47	2077.1	0.3	9.0	50	2094.4	0.2	35.9	32	2633.8	0.6	0.8
Subtotal (MÖ)		90				42								50				56							
36	20	10	8	1533.4	1.2	8	1071.4	0.7	9.9	9	785.5	0.6	7.6	8	850.1	0.4	13.4	8	859.2	0.3	37.5	8	1349.0	1.0	1.1
	30	10	3	2656.0	0.8	6	1847.1	0.1	9.5	9	1019.0	0.1	7.5	9	928.0	0.1	13.2	9	955.6	0.1	36.0	6	2239.6	0.6	1.3
	40	10	1	3519.4	2.8	3	3006.7	1.9	11.0	3	2857.3	1.1	7.8	4	2478.4	1.2	12.2	3	2997.4	1.5	36.7	0	*TL*	2.4	
	50	10	0	*TL*	3.4	2	3356.7	1.6		3	3350.2	1.5		3	3166.5	0.8		1	3501.0	1.1		0	*TL*	1.9	
	60	10	0	*TL*	4.9	0	*TL*	2.6		0	*TL*	2.6		0	*TL*	1.7		1	3537.9	2.5		0	*TL*	3.6	
	70	10	0	*TL*	6.3	0	*TL*	3.8		0	*TL*	2.6		0	*TL*	1.8		0	*TL*	2.6		0	*TL*	5.3	
	80	10	0	*TL*	11.4	0	*TL*	5.0		0	*TL*	3.5		0	*TL*	3.1		0	*TL*	3.9		0	*TL*	4.2	
	90	10	0	*TL*	19.5	0	*TL*	5.2		0	*TL*	3.2		0	*TL*	3.9		0	*TL*	4.1		0	*TL*	7.3	
	100	10	0	*TL*	26.2	0	*TL*	4.7		0	*TL*	4.7		0	*TL*	4.2		0	*TL*	4.6		0	*TL*	7.0	
Subtotal		90	12	3256.5	8.5	19	3031.3	2.8	9.9	24	2890.2	2.2	7.6	24	2824.8	1.9	13.2	22	2916.8	2.3	37.1	14	3198.7	3.7	1.1
Subtotal (MÖ)		90				10								17				14							
48	20	10	3	3066.4	4.6	5	2399.2	3.8	7.4	6	1964.7	1.9	11.8	7	1627.1	0.7	20.3	8	1405.8	0.5	38.0	5	2567.3	4.3	0.2
	30	10	0	*TL*	6.3	0	*TL*	4.1		1	3322.5	3.4		3	3232.1	2.9		2	3225.3	2.8		0	*TL*	6.5	
	40	10	0	*TL*	8.1	2	3413.6	6.7		4	3323.9	2.3		3	2884.0	1.7		5	2912.6	2.5		0	*TL*	3.8	
	50	10	0	*TL*	7.8	0	*TL*	7.4		0	*TL*	6.0		0	*TL*	3.6		0	*TL*	6.6		0	*TL*	10.6	
	60	10	0	*TL*	11.1	0	*TL*	10.1		0	*TL*	10.3		0	*TL*	8.5		0	*TL*	8.5		0	*TL*	9.1	
	70	10	0	*TL*	14.9	0	*TL*	9.2		0	*TL*	9.6		0	*TL*	9.0		0	*TL*	7.9		0	*TL*	12.0	
	80	10	0	*TL*	30.9	0	*TL*	10.9		0	*TL*	9.7		0	*TL*	9.5		0	*TL*	10.0		0	*TL*	10.7	
	90	10	0	*TL*	53.7	0	*TL*	10.5		0	*TL*	9.8		0	*TL*	8.3		0	*TL*	9.8		0	*TL*	13.2	
	100	10	0	*TL*	67.3	0	*TL*	10.1		0	*TL*	9.2		0	*TL*	9.7		0	*TL*	8.7		0	*TL*	22.0	
Subtotal		90	3	3540.7	22.8	7	3445.9	8.1	7.4	11	3356.8	6.9	11.8	13	3260.4	6.0	20.3	15	3238.2	6.3	38.0	5	3485.3	10.2	0.2
Subtotal (MÖ)		90				3								4				5							
Total		270	36	3264.6	10.8	75	2860.8	3.7	11.4	76	2876.0	3.1	6.5	84	2720.7	2.7	11.4	87	2749.8	3.0	36.4	51	3105.9	4.9	0.8
Total (MÖ)		270				55								71				75							

Also here, a comparison of the six routing policies reveals that policies with a larger state space regarding the number of feasible transitions are more difficult to solve with the BPC algorithm. The extreme cases are 36 optima for optimal and 87 optima for return.

Comparing the policies with respect to the total cost, our results are again in line with those of Petersen (1997) for the SPRP for the warehouse shape $m : C = 1 : 1$. Sorting the policies in relative difference Δ_{Pol} yields optimal, composite (with Δ_{Pol}= 0.8), largest gap, midpoint, traversal, and return (with Δ_{Pol}= 36.4). Depending on the policy, Δ_{Pol} either increases with increasing picker capacity Q (largest gap and midpoint), decreases (traversal and composite), or is stable (return).

6. Conclusions and Outlook

We have introduced a new approach to exactly solve two-level problems in warehousing in which one level concerns picking operations in manual picker-to-parts warehouses. On the one hand, we addressed the SPRP-SS, in which the higher-level decisions are the selection of pick positions for requested articles that are collectible from, in general, more than one pick position per article. On the other hand, we considered the JOBPRP, in which the higher-level decision is the grouping of customer orders (comprising the collection of one or several articles) into batches. In both cases, the modeling of the lower-level picker routing decisions is inspired by the dynamic program of Ratliff and Rosenthal (1983). To properly cope with the selection or grouping aspect, we extended the state space of Ratliff and Rosenthal. Since solving a dynamic program over the extended state space does not result in a feasible solution, we add consistency constraints and propose to solve the resulting picker-routing problems with the help of a MIP solver.

Our work shares some similarities with recent approaches that exploit the fact that pseudo-polynomial formulations exist for several fundamental problems (de Lima *et al.*, 2022). For example, in cutting and packing, a feasible structure, i.e., a cutting pattern or packing of items, is in one-to-one correspondence with a path in the underlying state space over which the pseudo-polynomial formulation is defined. Solving these formulations with a MIP solver has become possible even for large instances, either directly or by nesting optimal solutions with (a sequence of) restrictions and relaxations of the respective pseudo-polynomial formulation. Our approach is, however, different as we do not directly use the original state space. We first extend the state space, then add consistency constraints, and finally solve the resulting pure binary formulation using a MIP solver. For the SPRP-SS, the model must be solved only once, while for the JOBPRP, we obtain a formulation of the pricing subproblem that must be solved multiple times within a BPC algorithm.

The new modeling and solution approach has several advantages:

- We have presented four binary formulations, one for the SPRP-SS, one for the SPRP with profits, and two for the JOBPRP (a pure binary compact model and an extensive path-based formulation, where the latter was known as the set-partitioning formulation of the JOBPRP). The new formulations are simple and elegant in the sense that only very few types of variables and constraints are needed to entirely capture the respective problem (one may compare with the models of Goeke and Schneider, 2021; Bahçeci and Öncan, 2021; Scholz *et al.*, 2016).
- Our modeling approach is generic, since it applies to variants of the picker routing problem whenever a dynamic-programming approach is known for this variant. This includes different warehouse layouts, multiple possible endpoints of a picker tour, and other extensions (Masae *et al.*, 2020b; Löffler *et al.*, 2022).
- All exact solution approaches for the SPRP-SS use integer programming formulations solved directly with a MIP solver. The most competitive one is the recent and remarkably well-performing formulation of Goeke and Schneider (2021), for which the associated article received the *Meritorious Paper Award* of the journal *INFORMS Journal on Computing*. Our new network flow-based formulation for the SPRP-SS mostly outperforms it with an overall average speedup ratio of 3.6.
- Our BPC algorithm for the JOBPRP is the first column generation-based approach that exactly solves general instances (not only unit-weight instances) with routing policy optimal. A modified version of our BPC algorithm can also exactly solve instances with the policies traversal, midpoint, return, largest gap, and composite.

- The presented BPC algorithm for the JOBPRP has a convincing performance: (i) The entire benchmark set of Bahçeci and Öncan (2021) is solved optimally with an average computation time of less than one second per instance. (ii) 286 new best solutions have been computed for the benchmark set of Henn *et al.* (2012). Only metaheuristics have been applied to these instances in the past. (iii) For the policies traversal, midpoint, and return, at least 22, 21, and 20 instances of the benchmark set of Muter and Öncan (2015) have been solved to optimality for the first time, respectively.

Other integrated operational planning problems in warehousing that include a picker routing component (see van Gils *et al.*, 2018) may benefit from our modeling and solution method as well. Examples of two- or multi-level optimization problems are storage assignment (Petersen and Schmenner, 1999), order batching and sequencing (Menéndez *et al.*, 2017), and JOBPRP with scatter storage. The last one is the synthesis of the two problems investigated in this work. This natural extension gives rise to an integrated three-level optimization problem of grouping orders, selecting pick positions, and routing pickers.

In the future, we expect further methodological improvements making it possible to attack JOBPRP instances of very large size that please the planning needs of the big e-commerce companies. Such significant improvements in column generation-based methods are possible and have been made in the past for vehicle-, flight-, and crew-scheduling applications in public transit and the airline industries (Desaulniers and Hickman, 2007; Barnhart *et al.*, 2003).

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Appendix A. Instance Generation Procedure for SPRP-SS Instances

First, the generation procedure described by Weidinger and Goeke and Schneider completely fills the warehouse with articles. The assumption is that one article is stored in each of the $m \cdot C$ possible pick positions. In particular, opposite cells in an aisle are considered as one pick position. The scatter factor α can be realized by defining a set of $\xi = \lceil mC/\alpha \rceil \geq n$ different articles stored in the warehouse (not all of them are in the final pick list). To ensure that exactly ξ different articles are available, each of them is randomly assigned to a unique pick position.

Second, all ξ different articles are divided into the three classes A, B, and C (in practice depending on the turnover rate) with 20% in class A, 30% in class B, and 50% in class C. The ABC class-based addition of further SKUs must now ensure that (on average) 80% of the positions are stocked with A-class articles, 15% with B-class articles, and 5% with C-class articles. One by one, articles are assigned to positions as follows: According to the 80-15-5 distribution, a class is chosen. Then, a random article of that class is selected. Finally, this article is assigned randomly to one of the (up to this point) free storage positions in the warehouse.

Third, supply values are then determined with a further distinction between instances with

- *sufficiently many items* or *unit demand*: Here, the demand and supply values are set to $d_s = b_{sp} = 1$ for all $s \in S$ and $p \in P_s$;
- *varying supply and varying demand*: Here, the supply available at a position is randomly drawn as $b_{sp} \in \{1, 2, 3\}$. Moreover, demands are randomly drawn as $d_s \in \{1, \ldots, \min(6, \sum_p b_{sp})\}$.

Finally, the pick list is generated so that it contains exactly n different articles. Iteratively, a pick position is randomly drawn. If the SKU at this position is not yet an article in the pick list, this article s is chosen to be included in the pick list.

Appendix B. Results for SPRP-SS Instances with varying supply and varying demand.

For the sake of brevity, Table B.8 has been moved to the Appendix. It is structurally identical to Table 3 but for varying supply and varying demand.

Table B.8: Comparison of GS and NF with computation times (in milliseconds) of GS and speedup factor comparing GS and NF for the SPRP-SS and varying supply and varying demand.

Scatter factor	Warehouse dimension	Number of SKUs in pick list $a = 3$		$a = 7$		$a = 15$		$a = 30$	
	(m, C)	t_{NF}	t_{GS}/t_{NF}	t_{NF}	t_{GS}/t_{NF}	t_{NF}	t_{GS}/t_{NF}	t_{NF}	t_{GS}/t_{NF}
$\alpha = 2$	$(5, 30)$	46.8	4.5	67.0	2.0	87.9	1.1	107.9	0.9
	$(5, 60)$	47.4	5.1	56.1	2.7	99.0	1.2	127.2	0.9
	$(5, 180)$	59.4	6.6	71.8	2.3	106.7	1.0	167.1	0.8
	$(10, 30)$	70.8	5.4	92.7	2.3	189.7	1.4	231.5	1.0
	$(10, 60)$	56.7	9.6	78.9	2.9	159.4	1.6	309.2	0.9
	$(10, 180)$	71.4	6.9	87.8	2.7	147.4	1.3	344.5	0.8
	$(25, 30)$	78.6	5.3	122.6	3.4	296.5	2.0	712.3	1.4
	$(25, 60)$	87.1	8.6	134.1	3.9	268.7	2.1	589.2	1.3
	$(25, 180)$	111.7	8.8	113.4	6.2	300.0	2.1	643.4	1.5
	$(50, 30)$	162.9	4.6	226.6	5.8	415.8	3.1	847.3	1.9
	$(50, 60)$	146.6	7.2	228.4	7.1	387.1	3.3	895.1	2.2
	$(50, 180)$	199.1	16.1	214.5	12.5	416.0	4.5	931.3	2.5
$\alpha = 5$	$(10, 30)$	226.4	3.4	505.8	1.5	794.4	0.8	1021.3	0.6
	$(10, 60)$	173.6	4.8	448.6	2.4	859.4	1.0	2183.4	0.5
	$(10, 180)$	134.0	5.5	396.8	3.0	1070.6	1.1	2980.6	0.5
	$(25, 30)$	558.2	4.5	1190.9	2.8	1913.9	1.1	4141.6	0.6
	$(25, 60)$	552.2	4.7	1229.1	2.7	2210.2	1.4	4174.6	0.6
	$(25, 180)$	439.6	7.7	912.3	4.5	1372.5	1.8	4862.5	0.7
	$(50, 30)$	667.3	4.2	1186.1	2.9	1369.5	2.8	4398.7	1.3
	$(50, 60)$	887.7	6.3	1021.8	5.1	1865.1	3.4	5889.4	1.3
	$(50, 180)$	461.9	8.6	964.9	9.3	1755.8	4.4	5620.5	1.6
$\alpha = 10$	$(25, 30)$	2022.4	4.0	3635.7	1.5	6265.0	0.7	8669.6	0.6
	$(25, 60)$	1657.2	3.2	6333.0	1.7	9376.2	0.9	10 845.0	0.6
	$(25, 180)$	914.0	5.2	3534.5	2.5	9228.1	1.5	13 239.4	0.8
	$(50, 30)$	3643.5	1.4	3959.6	2.3	10 193.5	1.1	17 616.0	0.8
	$(50, 60)$	2307.5	3.5	3425.4	3.3	9315.7	1.3	17 902.1	0.9
	$(50, 180)$	1599.1	4.8	4260.0	6.1	11 015.0	2.7	20 612.2	1.7

Appendix C. Results for Instances of Henn *et al.*

For the sake of brevity, Table C.9 has been moved to the Appendix. It is structurally identical to Table C.9 but for the storage policy CBDS.

Table C.9: Results for the benchmark set of Henn *et al.* with storage location assignment CBDS.

Instances			Policy										
			optimal			traversal				largest gap			
Q	$\|O\|$	#inst	#opt	time $\bar{t}$	gap	#opt	time $\bar{t}$	gap	Δ_{Pol}	#opt	time $\bar{t}$	gap	Δ_{Pol}
30	20	20	20	12.7	0.0	20	1.2	0.0	20.5	20	2.1	0.0	5.5
	30	20	20	32.3	0.0	20	5.3	0.0	19.7	20	6.2	0.0	5.0
	40	20	20	74.3	0.0	20	11.3	0.0	19.1	20	13.1	0.0	5.2
	50	20	20	225.9	0.0	20	17.9	0.0	19.4	20	44.6	0.0	5.3
	60	20	20	404.6	0.0	19	189.3	<0.1	19.7	20	245.0	0.0	5.4
	70	20	19	557.6	<0.1	20	39.8	0.0	20.2	20	250.8	0.0	5.5
	80	20	14	1484.3	<0.1	18	452.7	<0.1	19.2	18	1016.6	<0.1	5.4
	90	20	15	1582.6	<0.1	17	633.9	<0.1	18.7	16	1654.1	<0.1	5.6
	100	20	12	1960.3	0.1	18	513.4	<0.1	19.5	14	1701.1	<0.1	5.4
45	20	20	20	312.9	0.0	20	36.3	0.0	12.3	20	54.6	0.0	7.7
	30	20	16	1091.1	0.1	19	507.8	0.1	12.8	18	529.3	<0.1	7.4
	40	20	9	2766.1	0.2	17	931.1	0.0	12.8	17	1071.1	<0.1	7.4
	50	20	2	3476.4	0.3	16	963.3	0.0	13.7	15	1973.8	<0.1	6.8
	60	20	0	*TL*	0.5	12	1829.6	0.1		5	3128.9	0.1	
	70	20	0	*TL*	2.8	10	2572.1	0.2		0	*TL*	0.8	
	80	20	0	*TL*	5.2	3	3245.3	0.1		0	*TL*	2.0	
	90	20	0	*TL*	11.3	4	3190.2	0.1		0	*TL*	2.3	
	100	20	0	*TL*	13.7	1	3521.0	0.2		0	*TL*	5.3	
60	20	20	13	1655.6	0.4	17	929.6	0.3	10.0	18	666.1	0.2	9.1
	30	20	4	3226.7	0.7	11	1812.3	0.5	9.3	14	1530.1	0.3	9.8
	40	20	2	3450.6	1.5	7	2550.0	0.6	10.6	6	3107.9	0.4	10.6
	50	20	0	*TL*	1.9	7	2668.9	0.3		0	*TL*	0.6	
	60	20	0	*TL*	6.5	7	3115.9	0.3		0	*TL*	2.5	
	70	20	0	*TL*	12.1	2	3424.8	0.5		0	*TL*	4.4	
	80	20	0	*TL*	21.6	0	*TL*	0.7		0	*TL*	9.3	
	90	20	0	*TL*	26.5	0	*TL*	1.8		0	*TL*	16.3	
	100	20	0	*TL*	32.6	0	*TL*	2.2		0	*TL*	24.0	
75	20	20	12	2330.0	0.2	16	1073.2	0.4	7.3	19	543.2	<0.1	13.0
	30	20	6	3189.0	2.2	8	2336.5	2.2	8.8	12	2212.7	0.9	12.6
	40	20	0	*TL*	5.2	6	2776.8	1.7		3	3273.2	1.0	
	50	20	0	*TL*	5.6	3	3264.1	1.6		0	*TL*	3.8	
	60	20	0	*TL*	15.7	0	*TL*	1.8		0	*TL*	4.2	
	70	20	0	*TL*	30.2	1	3562.8	3.6		0	*TL*	14.9	
	80	20	0	*TL*	44.9	0	*TL*	3.0		0	*TL*	30.6	
	90	20	0	*TL*	52.1	0	*TL*	3.8		0	*TL*	41.6	
	100	20	0	*TL*	60.9	0	*TL*	4.1		0	*TL*	54.5	
Total		720	244	2573.1	9.9	359	1982.7	0.8	16.7	315	2239.6	6.1	6.6

In this part of the Appendix, we present instance-by-instance results for the JOBPRP. The entries in the Tables C.10–D.12 have the following meaning:

- distr.: distribution; uniformly distributed SKUs (UDS) or class-based distributed SKUs (CBDS);
- type: instance type; largest gap (L.G.) or traversal=S-shape (S-S.);
- $|O|$: number of orders;
- Q: picker capacity;
- No.: instance number;
- UB: upper bound;
- LB: lower bound;
- time t: computation time in seconds; the time limit TL is 1 hour (3600 seconds).

We report results of the branch-price-and-cut algorithm proposed in this paper and of the adaptive large neighborhood search and tabu search (Žulj *et al.*, 2018), denoted by BPC algorithm and ŽKS, respectively. The ŽKS results are not available for all instances.

Table C.10: Detailed results for the instances of Henn *et al.*.

					Policy										
					optimal			largest gap				traversal			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	L.G.	20	30	0	3077	3077	28.4	3231	3231	10.3		3760	3760	10.5	
CBDS	L.G.	20	30	1	3168	3168	4.4	3416	3416	1.3		3972	3972	0.5	
CBDS	L.G.	20	30	2	3509	3509	3.5	3737	3737	0.5		4462	4462	0.4	
CBDS	L.G.	20	30	3	2814	2814	48.4	2949	2949	7.5		3230	3230	0.9	
CBDS	L.G.	20	30	4	2763	2763	7.4	2885	2885	1.8		3301	3301	1.5	
CBDS	L.G.	20	30	5	3538	3538	4.3	3807	3807	0.8		4289	4289	0.3	
CBDS	L.G.	20	30	6	3314	3314	2.8	3428	3428	0.6		4100	4100	0.5	
CBDS	L.G.	20	30	7	3780	3780	10.0	3942	3942	1.1		4474	4474	0.3	
CBDS	L.G.	20	30	8	4127	4127	2.7	4366	4366	0.7		4916	4916	0.3	
CBDS	L.G.	20	30	9	4158	4158	6.5	4502	4502	0.9		5095	5095	0.6	
CBDS	L.G.	20	45	0	2362	2362	24.4	2572	2572	18.0		2701	2701	7.2	
CBDS	L.G.	20	45	1	2485	2485	664.9	2611	2611	82.4		2815	2815	30.5	
CBDS	L.G.	20	45	2	2195	2195	56.3	2364	2364	3.4		2488	2488	0.9	
CBDS	L.G.	20	45	3	2196	2196	234.1	2382	2382	26.2		2397	2397	3.8	
CBDS	L.G.	20	45	4	2201	2201	45.4	2376	2376	20.8		2494	2494	11.1	
CBDS	L.G.	20	45	5	2152	2152	27.7	2278	2278	35.1		2422	2422	1.9	
CBDS	L.G.	20	45	6	2544	2544	817.0	2769	2769	107.2		2793	2793	241.2	
CBDS	L.G.	20	45	7	2121	2121	147.0	2283	2283	18.0		2369	2369	12.2	
CBDS	L.G.	20	45	8	2629	2629	213.4	2761	2761	14.6		2970	2970	12.7	
CBDS	L.G.	20	45	9	2604	2604	219.4	2757	2757	4.5		2876	2876	31.4	
CBDS	L.G.	20	60	0	1860	1860	283.6	2014	2014	114.5		2028	2028	26.3	
CBDS	L.G.	20	60	1	1844	1828	*TL*	2019	2019	1431.8		2107	2054	*TL*	
CBDS	L.G.	20	60	2	1903	1903	348.3	2073	2073	33.7		2079	2079	53.4	
CBDS	L.G.	20	60	3	1744	1744	226.6	1899	1899	74.9		1927	1927	210.0	
CBDS	L.G.	20	60	4	1848	1817	*TL*	2015	2001	*TL*		1981	1967	*TL*	
CBDS	L.G.	20	60	5	1878	1878	2625.7	2048	2048	101.2		2081	2081	218.2	
CBDS	L.G.	20	60	6	1777	1777	577.5	1930	1930	32.8		2066	2066	854.4	
CBDS	L.G.	20	60	7	1784	1784	30.6	1980	1980	356.0		1965	1965	385.1	
CBDS	L.G.	20	60	8	1980	1980	915.6	2142	2142	16.6		2129	2129	7.0	
CBDS	L.G.	20	60	9	1974	1974	170.0	2163	2163	98.7		2166	2166	35.4	
CBDS	L.G.	20	75	0	1612	1607	*TL*	1802	1802	924.1		1760	1741	*TL*	
CBDS	L.G.	20	75	1	2001	2001	2406.2	2275	2275	128.3		2179	2179	52.5	
CBDS	L.G.	20	75	2	1637	1637	73.0	1838	1838	6.9		1748	1748	143.3	
CBDS	L.G.	20	75	3	1496	1496	3334.5	1659	1659	807.8		1635	1635	570.7	
CBDS	L.G.	20	75	4	1191	1191	2060.7	1341	1341	243.9		1283	1283	2644.6	
CBDS	L.G.	20	75	5	1664	1664	53.7	1855	1855	71.6		1802	1802	45.8	
CBDS	L.G.	20	75	6	1469	1463	*TL*	1673	1673	334.1		1552	1552	598.2	
CBDS	L.G.	20	75	7	1796	1796	3285.6	1986	1986	125.7		1987	1987	190.7	
CBDS	L.G.	20	75	8	1390	1382	*TL*	1529	1529	1036.5		1601	1539	*TL*	
CBDS	L.G.	20	75	9	1461	1461	*TL*	1688	1678	*TL*		1641	1625	*TL*	
CBDS	L.G.	30	30	0	4557	4557	20.0	4794	4794	3.6		5446	5446	1.2	
CBDS	L.G.	30	30	1	4425	4425	24.6	4614	4614	5.3		5340	5340	37.7	
CBDS	L.G.	30	30	2	4807	4807	95.8	5070	5070	24.3		5695	5695	4.2	
CBDS	L.G.	30	30	3	5767	5767	12.0	6079	6079	2.7		6539	6539	0.9	

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Instance					Policy										
					optimal			largest gap				traversal			
					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	L.G.	30	30	4	6393	6393	4.5	6774	6774	1.4		7637	7637	0.4	
CBDS	L.G.	30	30	5	6311	6311	12.0	6658	6658	8.8		7594	7594	1.0	
CBDS	L.G.	30	30	6	4633	4633	167.9	4806	4806	3.3		5430	5430	1.2	
CBDS	L.G.	30	30	7	5131	5131	6.5	5408	5408	0.9		6207	6207	0.7	
CBDS	L.G.	30	30	8	4531	4531	29.3	4644	4644	3.5		5466	5466	31.2	
CBDS	L.G.	30	30	9	4457	4457	117.3	4668	4668	23.9		5282	5282	8.8	
CBDS	L.G.	30	45	0	3743	3743	255.3	4048	4048	102.3		4240	4240	6.6	
CBDS	L.G.	30	45	1	3611	3611	361.6	3904	3904	23.7		4031	4031	13.0	
CBDS	L.G.	30	45	2	3429	3373	*TL*	3641	3619	*TL*		3822	3788	*TL*	
CBDS	L.G.	30	45	3	3673	3673	275.8	3935	3935	55.1		4111	4111	36.7	
CBDS	L.G.	30	45	4	3772	3772	124.3	4089	4089	53.3		4196	4196	2.1	
CBDS	L.G.	30	45	5	3787	3787	581.5	4001	4001	7.9		4186	4186	44.8	
CBDS	L.G.	30	45	6	3499	3499	53.2	3775	3775	28.7		4063	4063	64.6	
CBDS	L.G.	30	45	7	3310	3310	225.7	3547	3547	130.7		3720	3720	19.4	
CBDS	L.G.	30	45	8	3632	3632	280.7	3796	3796	62.5		4128	4128	25.2	
CBDS	L.G.	30	45	9	3953	3953	644.6	4342	4342	79.3		4428	4428	6.8	
CBDS	L.G.	30	60	0	2883	2856	*TL*	3217	3188	*TL*		3189	3153	*TL*	
CBDS	L.G.	30	60	1	2859	2827	*TL*	3191	3169	*TL*		3124	3080	*TL*	
CBDS	L.G.	30	60	2	2620	2619	*TL*	2891	2891	733.2		2851	2851	699.6	
CBDS	L.G.	30	60	3	2655	2655	2093.2	2855	2855	57.6		2881	2881	58.4	
CBDS	L.G.	30	60	4	3025	3004	*TL*	3303	3303	544.7		3268	3261	*TL*	
CBDS	L.G.	30	60	5	3280	3250	*TL*	3575	3554	*TL*		3710	3651	*TL*	
CBDS	L.G.	30	60	6	2598	2580	*TL*	2773	2773	1053.4		2864	2855	*TL*	
CBDS	L.G.	30	60	7	2914	2905	*TL*	3175	3175	405.6		3196	3195	*TL*	
CBDS	L.G.	30	60	8	2742	2732	*TL*	2971	2971	2138.1		2896	2896	7.7	
CBDS	L.G.	30	60	9	2654	2654	1241.8	2929	2929	268.4		2921	2921	78.0	
CBDS	L.G.	30	75	0	2364	2278	*TL*	2634	2580	*TL*		2547	2475	*TL*	
CBDS	L.G.	30	75	1	1966	1925	*TL*	2175	2168	*TL*		2069	2053	*TL*	
CBDS	L.G.	30	75	2	2428	2408	*TL*	2747	2747	2318.2		2578	2560	*TL*	
CBDS	L.G.	30	75	3	2332	2292	*TL*	2561	2556	*TL*		2472	2472	379.6	
CBDS	L.G.	30	75	4	2303	2303	674.3	2643	2643	1047.0		2561	2561	188.1	
CBDS	L.G.	30	75	5	2381	2381	2538.5	2651	2651	487.5		2550	2550	48.1	
CBDS	L.G.	30	75	6	2384	2384	1375.8	2694	2694	1515.0		2650	2616	*TL*	
CBDS	L.G.	30	75	7	2280	2117	*TL*	2542	2409	*TL*		2364	2253	*TL*	
CBDS	L.G.	30	75	8	2222	2038	*TL*	2496	2288	*TL*		2421	2235	*TL*	
CBDS	L.G.	30	75	9	2174	2020	*TL*	2219	2219	1758.2		2337	2169	*TL*	
CBDS	L.G.	40	30	0	6985	6985	57.5	7300	7300	13.2	7300	8167	8167	31.1	
CBDS	L.G.	40	30	1	6929	6929	31.8	7188	7188	9.0	7188	8507	8507	0.9	
CBDS	L.G.	40	30	2	5837	5837	189.1	6190	6190	4.6	6190	6656	6656	5.4	
CBDS	L.G.	40	30	3	6003	6003	301.0	6239	6239	4.7	6239	7142	7142	19.7	
CBDS	L.G.	40	30	4	6752	6752	28.0	7181	7181	18.2	7181	8101	8101	1.2	
CBDS	L.G.	40	30	5	6948	6948	33.5	**7279**	7279	2.2	7345	8155	8155	1.0	
CBDS	L.G.	40	30	6	6342	6342	176.7	6510	6510	60.8	6510	7557	7557	130.2	
CBDS	L.G.	40	30	7	5714	5714	101.9	5983	5983	32.3	5983	6673	6673	12.5	
CBDS	L.G.	40	30	8	7187	7187	13.1	7640	7640	3.6	7640	8588	8588	0.6	
CBDS	L.G.	40	30	9	6126	6126	111.4	6470	6470	9.0	6470	7333	7333	5.8	
CBDS	L.G.	40	45	0	5324	5305	*TL*	5592	5591	*TL*	5592	5964	5964	61.8	
CBDS	L.G.	40	45	1	5182	5172	*TL*	5532	5532	865.9	5532	5834	5834	302.3	
CBDS	L.G.	40	45	2	4625	4607	*TL*	4924	4924	3229.1	4924	5165	5165	1204.5	
CBDS	L.G.	40	45	3	4428	4420	*TL*	**4792**	4792	238.2	4793	4852	4852	115.0	
CBDS	L.G.	40	45	4	4737	4737	2249.7	**5111**	5111	526.5	5120	5336	5336	843.4	
CBDS	L.G.	40	45	5	4076	4076	2498.5	**4432**	4432	276.4	4437	4666	4666	717.6	
CBDS	L.G.	40	45	6	5011	5007	*TL*	5339	5339	215.8	5339	5644	5644	457.2	
CBDS	L.G.	40	45	7	4542	4539	*TL*	4912	4912	537.5	4912	4952	4952	41.3	
CBDS	L.G.	40	45	8	5233	5233	926.0	5501	5501	267.9	5501	5867	5867	40.6	
CBDS	L.G.	40	45	9	4084	4084	2424.3	4296	4296	786.8	4296	4721	4721	2641.1	
CBDS	L.G.	40	60	0	3681	3615	*TL*	3997	3958	*TL*	3997	3993	3929	*TL*	
CBDS	L.G.	40	60	1	3158	3093	*TL*	**3397**	3368	*TL*	3406	3457	3454	*TL*	
CBDS	L.G.	40	60	2	3528	3486	*TL*	**3904**	3902	*TL*	3927	3752	3752	550.3	
CBDS	L.G.	40	60	3	3790	3719	*TL*	4152	4120	*TL*	**4149**	4046	4008	*TL*	
CBDS	L.G.	40	60	4	3860	3860	3241.6	**4269**	4269	1284.1	4360	4270	4270	146.2	
CBDS	L.G.	40	60	5	3631	3592	*TL*	**3971**	3971	1671.3	3981	3875	3875	2268.6	
CBDS	L.G.	40	60	6	3640	3627	*TL*	**3970**	3962	*TL*	3974	3918	3914	*TL*	
CBDS	L.G.	40	60	7	3821	3802	*TL*	**4175**	4175	1902.7	4190	4140	4140	104.1	
CBDS	L.G.	40	60	8	3806	3654	*TL*	4173	4048	*TL*	**4114**	3983	3879	*TL*	
CBDS	L.G.	40	60	9	3969	3942	*TL*	**4328**	4327	*TL*	4340	4274	4259	*TL*	
CBDS	L.G.	40	75	0	3105	3013	*TL*	3454	3408	*TL*	**3434**	3266	3252	*TL*	
CBDS	L.G.	40	75	1	3345	3090	*TL*	3446	3446	1535.6	3446	3393	3316	*TL*	
CBDS	L.G.	40	75	2	2974	2908	*TL*	3366	3313	*TL*	**3350**	3224	3177	*TL*	
CBDS	L.G.	40	75	3	3017	2702	*TL*	**2978**	2968	*TL*	2990	2882	2882	962.9	

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Instance					Policy										
					optimal			largest gap				traversal			
					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\lvert O\rvert$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	L.G.	40	75	4	3051	2955	TL	3365	3329	TL	**3360**	3188	3138	TL	
CBDS	L.G.	40	75	5	3419	3149	TL	**3502**	3490	TL	3568	3424	3424	546.7	
CBDS	L.G.	40	75	6	2617	2502	TL	**2841**	2812	TL	2848	2886	2729	TL	
CBDS	L.G.	40	75	7	2994	2905	TL	3259	3209	TL	**3243**	3244	3179	TL	
CBDS	L.G.	40	75	8	3263	3170	TL	3646	3608	TL	**3614**	3364	3364	1807.0	
CBDS	L.G.	40	75	9	3059	2952	TL	3417	3333	TL	**3384**	3332	3191	TL	
CBDS	L.G.	50	30	0	9595	9595	12.8	10052	10052	2.7		11858	11858	1.0	
CBDS	L.G.	50	30	1	7693	7693	192.5	8131	8131	46.2		9272	9272	10.6	
CBDS	L.G.	50	30	2	8963	8963	31.2	9485	9485	9.3		11116	11116	1.9	
CBDS	L.G.	50	30	3	8002	8002	1503.4	8320	8320	373.0		9394	9394	16.8	
CBDS	L.G.	50	30	4	7639	7639	222.4	8073	8073	9.0		8976	8976	6.4	
CBDS	L.G.	50	30	5	9782	9782	11.6	10246	10246	2.2		11466	11466	1.0	
CBDS	L.G.	50	30	6	8012	8012	731.0	8478	8478	112.7		9464	9464	113.8	
CBDS	L.G.	50	30	7	9277	9277	11.4	9857	9857	11.6		11542	11542	1.1	
CBDS	L.G.	50	30	8	9121	9121	26.8	9484	9484	3.1		10515	10515	0.9	
CBDS	L.G.	50	30	9	8216	8216	205.2	8641	8641	46.9		9903	9903	93.8	
CBDS	L.G.	50	45	0	5607	5607	2656.2	5987	5987	814.7		6413	6413	198.8	
CBDS	L.G.	50	45	1	5219	5195	TL	5607	5603	TL		5750	5747	TL	
CBDS	L.G.	50	45	2	5052	5022	TL	5392	5392	3512.7		5658	5647	TL	
CBDS	L.G.	50	45	3	5286	5264	TL	5797	5797	2833.7		5933	5929	TL	
CBDS	L.G.	50	45	4	5437	5424	TL	5765	5763	TL		6042	6042	677.1	
CBDS	L.G.	50	45	5	6139	6126	TL	6568	6568	1098.9		6798	6798	109.4	
CBDS	L.G.	50	45	6	5932	5911	TL	6389	6389	969.0		6575	6575	372.0	
CBDS	L.G.	50	45	7	5268	5228	TL	5624	5607	TL		5772	5764	TL	
CBDS	L.G.	50	45	8	5572	5538	TL	5986	5986	2391.6		6194	6194	384.5	
CBDS	L.G.	50	45	9	5740	5734	TL	6213	6213	307.4		6287	6287	79.3	
CBDS	L.G.	50	60	0	5124	4795	TL	5366	5335	TL		5218	5198	TL	
CBDS	L.G.	50	60	1	4102	4038	TL	4371	4362	TL		4498	4437	TL	
CBDS	L.G.	50	60	2	4103	3971	TL	4385	4377	TL		4407	4407	TL	
CBDS	L.G.	50	60	3	4281	4246	TL	4723	4668	TL		4548	4545	TL	
CBDS	L.G.	50	60	4	4431	4407	TL	4897	4873	TL		4794	4794	630.6	
CBDS	L.G.	50	60	5	4271	4223	TL	4635	4635	TL		4604	4600	TL	
CBDS	L.G.	50	60	6	4058	4032	TL	4654	4453	TL		4357	4357	1163.2	
CBDS	L.G.	50	60	7	3958	3942	TL	4343	4335	TL		4341	4341	795.2	
CBDS	L.G.	50	60	8	4774	4659	TL	5080	5077	TL		5103	5103	934.9	
CBDS	L.G.	50	60	9	4425	4369	TL	4804	4803	TL		4856	4820	TL	
CBDS	L.G.	50	75	0	3944	3580	TL	4181	3240	TL		3824	3824	TL	
CBDS	L.G.	50	75	1	3823	3771	TL	4226	4184	TL		4143	4097	TL	
CBDS	L.G.	50	75	2	3618	3406	TL	3829	3800	TL		3662	3647	TL	
CBDS	L.G.	50	75	3	3802	3689	TL	4156	4080	TL		4040	3915	TL	
CBDS	L.G.	50	75	4	3711	3530	TL	4033	3928	TL		3934	3794	TL	
CBDS	L.G.	50	75	5	3367	3171	TL	3723	3514	TL		3662	3440	TL	
CBDS	L.G.	50	75	6	4103	3778	TL	4375	4196	TL		4125	4125	1054.1	
CBDS	L.G.	50	75	7	3857	3734	TL	4355	4263	TL		3978	3919	TL	
CBDS	L.G.	50	75	8	3983	3741	TL	4585	4268	TL		3968	3968	1472.2	
CBDS	L.G.	50	75	9	4197	3878	TL	4556	4299	TL		4109	4109	1554.9	
CBDS	L.G.	60	30	0	9801	9801	189.2	**10469**	10469	81.2	10470	11688	11688	8.5	
CBDS	L.G.	60	30	1	10013	10013	51.5	**10502**	10502	39.9	10560	11817	11817	1.6	
CBDS	L.G.	60	30	2	11359	11359	34.4	11997	11997	14.5	11997	13668	13668	5.5	
CBDS	L.G.	60	30	3	9255	9255	1575.3	9724	9724	371.7	9724	10949	10949	16.5	
CBDS	L.G.	60	30	4	11609	11609	181.7	12255	12255	1390.1	12255	13652	13652	11.1	
CBDS	L.G.	60	30	5	9110	9110	602.8	**9564**	9564	416.7	9624	10828	10828	33.5	
CBDS	L.G.	60	30	6	11555	11555	14.7	**12221**	12221	3.5	12339	13914	13914	1.2	
CBDS	L.G.	60	30	7	9282	9282	162.2	**9765**	9765	119.4	9771	10837	10837	4.0	
CBDS	L.G.	60	30	8	9137	9137	245.3	9688	9688	92.6	9688	11097	11097	3.5	
CBDS	L.G.	60	30	9	10715	10715	13.2	**11259**	11259	3.4	11417	13120	13120	1.2	
CBDS	L.G.	60	45	0	7368	7342	TL	**7907**	7905	TL	7925	8288	8283	TL	
CBDS	L.G.	60	45	1	6652	6619	TL	**7068**	7049	TL	7094	7379	7356	TL	
CBDS	L.G.	60	45	2	6640	6591	TL	**7203**	7158	TL	7208	7393	7339	TL	
CBDS	L.G.	60	45	3	6106	6059	TL	**6560**	6532	TL	6566	6819	6801	TL	
CBDS	L.G.	60	45	4	6582	6553	TL	**7044**	7043	TL	7071	7290	7290	TL	
CBDS	L.G.	60	45	5	6532	6495	TL	**6912**	6912	881.8	6946	7256	7256	610.8	
CBDS	L.G.	60	45	6	6845	6710	TL	**7245**	7245	2876.5	7275	7640	7640	1305.1	
CBDS	L.G.	60	45	7	6413	6403	TL	**6840**	6824	TL	6951	7177	7177	465.6	
CBDS	L.G.	60	45	8	6849	6826	TL	**7383**	7379	TL	7423	7681	7669	TL	
CBDS	L.G.	60	45	9	7054	7028	TL	**7535**	7535	2028.5	7614	7948	7948	210.7	
CBDS	L.G.	60	60	0	5391	5141	TL	5910	5375	TL	**5861**	5802	5770	TL	
CBDS	L.G.	60	60	1	5465	5182	TL	**5783**	5725	TL	5852	5619	5619	2959.4	
CBDS	L.G.	60	60	2	5266	4905	TL	5446	5423	TL	**5427**	5484	5478	TL	
CBDS	L.G.	60	60	3	5767	5448	TL	6351	6099	TL	**6246**	6100	6083	TL	

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					Policy										
					optimal			largest gap				traversal			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	L.G.	60	60	4	5474	5383	TL	**6018**	5964	TL	6038	5830	5830	2626.1	
CBDS	L.G.	60	60	5	4988	4645	TL	**5279**	5265	TL	5286	5530	5471	TL	
CBDS	L.G.	60	60	6	5167	4969	TL	**5461**	5391	TL	5471	5439	5410	TL	
CBDS	L.G.	60	60	7	5455	5364	TL	5921	5840	TL	**5892**	5947	5918	TL	
CBDS	L.G.	60	60	8	5554	4659	TL	**5807**	5781	TL	5873	5681	5673	TL	
CBDS	L.G.	60	60	9	5024	4865	TL	5436	5381	TL	**5423**	5443	5411	TL	
CBDS	L.G.	60	75	0	4534	2938	TL	4933	4528	TL	**4616**	4413	4320	TL	
CBDS	L.G.	60	75	1	5108	4443	TL	5858	5188	TL	**5495**	5253	5157	TL	
CBDS	L.G.	60	75	2	4872	3665	TL	5358	5041	TL	**5112**	4884	4866	TL	
CBDS	L.G.	60	75	3	4564	4267	TL	5094	4920	TL	**5036**	4715	4697	TL	
CBDS	L.G.	60	75	4	4679	4140	TL	4989	4877	TL	**4968**	4835	4683	TL	
CBDS	L.G.	60	75	5	3671	3603	TL	4130	4087	TL	**4120**	3962	3902	TL	
CBDS	L.G.	60	75	6	4377	3914	TL	4785	4467	TL	**4503**	4464	4427	TL	
CBDS	L.G.	60	75	7	4822	4393	TL	5237	5163	TL	**5213**	5230	5167	TL	
CBDS	L.G.	60	75	8	4465	3851	TL	4936	4793	TL	**4890**	4551	4525	TL	
CBDS	L.G.	60	75	9	4400	3463	TL	4740	4594	TL	**4676**	4428	4379	TL	
CBDS	L.G.	70	30	0	10062	10061	TL	10510	10510	478.5		12103	12103	34.9	
CBDS	L.G.	70	30	1	11276	11276	396.8	11799	11799	167.5		13146	13146	26.3	
CBDS	L.G.	70	30	2	10709	10709	401.4	11362	11362	720.5		12736	12736	54.5	
CBDS	L.G.	70	30	3	10945	10945	218.9	11452	11452	465.6		13085	13085	60.0	
CBDS	L.G.	70	30	4	11453	11453	186.5	12232	12232	128.7		13609	13609	6.1	
CBDS	L.G.	70	30	5	12442	12442	168.7	13128	13128	22.1		15123	15123	23.3	
CBDS	L.G.	70	30	6	12050	12050	53.0	12697	12697	4.0		14611	14611	5.0	
CBDS	L.G.	70	30	7	12374	12374	29.2	12894	12894	4.8		15097	15097	1.7	
CBDS	L.G.	70	30	8	11473	11473	260.8	12176	12176	886.3		13812	13812	3.1	
CBDS	L.G.	70	30	9	11025	11025	965.9	11729	11729	19.0		13462	13462	1.6	
CBDS	L.G.	70	45	0	8015	7947	TL	8463	8440	TL		9224	9174	TL	
CBDS	L.G.	70	45	1	8617	8417	TL	9227	9105	TL		9329	9326	TL	
CBDS	L.G.	70	45	2	8329	8244	TL	8906	8851	TL		9484	9379	TL	
CBDS	L.G.	70	45	3	7368	6755	TL	7841	7838	TL		8091	8091	129.0	
CBDS	L.G.	70	45	4	7667	7627	TL	8229	8208	TL		8522	8522	1676.8	
CBDS	L.G.	70	45	5	8118	7899	TL	8678	8610	TL		8949	8949	409.6	
CBDS	L.G.	70	45	6	8035	7800	TL	8319	8311	TL		8725	8708	TL	
CBDS	L.G.	70	45	7	8326	8301	TL	8898	8881	TL		9303	9303	2987.8	
CBDS	L.G.	70	45	8	7743	7612	TL	8174	8163	TL		8651	8651	3512.0	
CBDS	L.G.	70	45	9	7503	6948	TL	7961	7938	TL		8646	8625	TL	
CBDS	L.G.	70	60	0	6449	5650	TL	7082	6755	TL		6701	6688	TL	
CBDS	L.G.	70	60	1	6773	5909	TL	7505	6787	TL		7125	7117	TL	
CBDS	L.G.	70	60	2	6351	5535	TL	6642	6558	TL		6561	6547	TL	
CBDS	L.G.	70	60	3	5791	4623	TL	6466	6014	TL		6259	6115	TL	
CBDS	L.G.	70	60	4	6617	5993	TL	7518	6735	TL		6846	6784	TL	
CBDS	L.G.	70	60	5	6397	5287	TL	6676	6597	TL		6623	6527	TL	
CBDS	L.G.	70	60	6	6543	5826	TL	7533	6923	TL		6914	6882	TL	
CBDS	L.G.	70	60	7	5904	5597	TL	6482	6370	TL		6377	6377	TL	
CBDS	L.G.	70	60	8	6753	6403	TL	7220	7005	TL		6912	6912	2275.4	
CBDS	L.G.	70	60	9	5769	5564	TL	6118	6028	TL		5954	5954	1419.7	
CBDS	L.G.	70	75	0	5106	3493	TL	5571	5067	TL		5090	5090	2855.6	
CBDS	L.G.	70	75	1	5150	4241	TL	5926	5420	TL		5563	5358	TL	
CBDS	L.G.	70	75	2	5109	3915	TL	5726	5001	TL		5441	5349	TL	
CBDS	L.G.	70	75	3	5509	4652	TL	5905	5446	TL		5759	5617	TL	
CBDS	L.G.	70	75	4	5188	4503	TL	5781	4794	TL		5552	5225	TL	
CBDS	L.G.	70	75	5	5051	3467	TL	5644	5075	TL		5065	5041	TL	
CBDS	L.G.	70	75	6	5104	3948	TL	5662	5021	TL		5508	5181	TL	
CBDS	L.G.	70	75	7	5085	3996	TL	5561	5133	TL		5341	5014	TL	
CBDS	L.G.	70	75	8	5182	4059	TL	5743	4822	TL		5218	5154	TL	
CBDS	L.G.	70	75	9	4856	4065	TL	5403	5070	TL		5046	4991	TL	
CBDS	L.G.	80	30	0	12998	12998	519.2	**13697**	13697	140.4	13795	15262	15262	28.6	
CBDS	L.G.	80	30	1	11307	11307	TL	**11804**	11804	428.0	11822	13305	13305	528.0	
CBDS	L.G.	80	30	2	12193	12193	TL	12767	12767	1093.6	12767	14407	14407	103.8	
CBDS	L.G.	80	30	3	13854	13854	320.2	**14582**	14582	66.6	14674	16571	16571	2.0	
CBDS	L.G.	80	30	4	12435	12402	TL	13130	13107	TL	13130	14550	14510	TL	
CBDS	L.G.	80	30	5	12451	12445	TL	**13133**	13132	TL	13134	14696	14687	TL	
CBDS	L.G.	80	30	6	12029	12029	1085.8	**12608**	12608	2014.9	12623	14383	14383	99.8	
CBDS	L.G.	80	30	7	12080	12080	432.0	**12727**	12727	2656.7	12778	14206	14206	58.6	
CBDS	L.G.	80	30	8	12790	12780	TL	**13517**	13517	639.1	13532	14863	14863	82.6	
CBDS	L.G.	80	30	9	13845	13845	339.1	14755	14755	985.3	14755	16110	16110	17.4	
CBDS	L.G.	80	45	0	8680	8308	TL	**9120**	9078	TL	9164	9460	9428	TL	
CBDS	L.G.	80	45	1	8760	8732	TL	9701	7996	TL	**9373**	9779	9757	TL	
CBDS	L.G.	80	45	2	8665	8200	TL	**9152**	9127	TL	9203	9731	9727	TL	
CBDS	L.G.	80	45	3	8664	8542	TL	**9299**	9276	TL	9357	9468	9453	TL	

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Instance					Policy										
					optimal			largest gap				traversal			
					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	L.G.	80	45	4	9441	9103	TL	**10102**	10070	TL	10141	10420	10420	1267.7	
CBDS	L.G.	80	45	5	9158	8375	TL	**9512**	9485	TL	9529	9894	9885	TL	
CBDS	L.G.	80	45	6	8739	8529	TL	**9254**	9249	TL	9341	9704	9700	TL	
CBDS	L.G.	80	45	7	9467	8741	TL	**9898**	9869	TL	9932	10645	10597	TL	
CBDS	L.G.	80	45	8	9009	7542	TL	9639	9114	TL	**9301**	9654	9614	TL	
CBDS	L.G.	80	45	9	8412	8258	TL	**8914**	8898	TL	8956	9466	9459	TL	
CBDS	L.G.	80	60	0	7145	6344	TL	7575	7353	TL	**7445**	7598	7575	TL	
CBDS	L.G.	80	60	1	6812	5737	TL	7439	7056	TL	**7415**	7336	7255	TL	
CBDS	L.G.	80	60	2	7703	6404	TL	8341	7483	TL	**8316**	8131	8110	TL	
CBDS	L.G.	80	60	3	6704	5598	TL	7237	6670	TL	**7137**	7185	7042	TL	
CBDS	L.G.	80	60	4	6858	5789	TL	7728	6942	TL	**7350**	7210	7163	TL	
CBDS	L.G.	80	60	5	7589	5965	TL	8568	7325	TL	**8023**	7823	7807	TL	
CBDS	L.G.	80	60	6	7138	5904	TL	7933	7321	TL	**7642**	7336	7309	TL	
CBDS	L.G.	80	60	7	7596	6052	TL	8253	7570	TL	**8174**	7894	7885	TL	
CBDS	L.G.	80	60	8	7937	6238	TL	8568	8014	TL	**8420**	8410	8397	TL	
CBDS	L.G.	80	60	9	7242	5674	TL	7791	7129	TL	**7750**	7490	7483	TL	
CBDS	L.G.	80	75	0	6000	4602	TL	6858	5767	TL	**6569**	6388	6336	TL	
CBDS	L.G.	80	75	1	5526	3884	TL	5962	4801	TL	**5797**	5822	5499	TL	
CBDS	L.G.	80	75	2	5993	4238	TL	6591	5046	TL	**6308**	6233	5897	TL	
CBDS	L.G.	80	75	3	6381	4418	TL	6793	5473	TL	**6682**	6434	6307	TL	
CBDS	L.G.	80	75	4	5365	4040	TL	6294	4944	TL	**5920**	5815	5406	TL	
CBDS	L.G.	80	75	5	6099	4620	TL	6450	5770	TL	**6408**	6216	6139	TL	
CBDS	L.G.	80	75	6	6405	4049	TL	7175	5284	TL	**6900**	6490	6397	TL	
CBDS	L.G.	80	75	7	6152	3815	TL	6624	3113	TL	**6490**	6347	6157	TL	
CBDS	L.G.	80	75	8	6276	3722	TL	6838	4402	TL	**6406**	6404	6131	TL	
CBDS	L.G.	80	75	9	6328	4611	TL	6996	5671	TL	**6741**	6515	6389	TL	
CBDS	L.G.	90	30	0	13195	13195	996.6	13888	13888	531.9		15803	15803	116.9	
CBDS	L.G.	90	30	1	15260	15260	712.6	16262	16262	792.3		18141	18141	16.7	
CBDS	L.G.	90	30	2	15370	15370	227.9	16122	16122	455.7		18416	18416	27.7	
CBDS	L.G.	90	30	3	14263	14263	1446.3	15026	15026	1541.8		16713	16713	66.5	
CBDS	L.G.	90	30	4	14446	14446	798.2	15277	15277	1437.0		17448	17430	TL	
CBDS	L.G.	90	30	5	13514	13505	TL	14286	14283	TL		16399	16393	TL	
CBDS	L.G.	90	30	6	12141	12113	TL	12577	12545	TL		14104	14104	1079.2	
CBDS	L.G.	90	30	7	13289	13289	TL	14015	14015	1159.0		15505	15505	36.2	
CBDS	L.G.	90	30	8	14330	14330	2140.7	15210	15210	945.9		16744	16744	88.2	
CBDS	L.G.	90	30	9	14786	14769	TL	15554	15554	2364.7		17495	17495	83.3	
CBDS	L.G.	90	45	0	8936	8396	TL	9546	9503	TL		10020	9990	TL	
CBDS	L.G.	90	45	1	8909	7941	TL	9722	9491	TL		9821	9821	3265.6	
CBDS	L.G.	90	45	2	9350	8338	TL	9866	9836	TL		10299	10275	TL	
CBDS	L.G.	90	45	3	9463	9013	TL	10186	10131	TL		10643	10643	TL	
CBDS	L.G.	90	45	4	10743	10019	TL	11457	10790	TL		11979	11926	TL	
CBDS	L.G.	90	45	5	9539	9119	TL	10541	10052	TL		10559	10539	TL	
CBDS	L.G.	90	45	6	9758	8139	TL	10285	10253	TL		10824	10817	TL	
CBDS	L.G.	90	45	7	9805	9476	TL	10418	10183	TL		10858	10858	1240.5	
CBDS	L.G.	90	45	8	9374	7945	TL	10028	9656	TL		10470	10443	TL	
CBDS	L.G.	90	45	9	10109	8705	TL	10726	10692	TL		11111	11108	TL	
CBDS	L.G.	90	60	0	7619	5794	TL	8282	6536	TL		8103	7964	TL	
CBDS	L.G.	90	60	1	7701	5943	TL	8103	7686	TL		8154	8065	TL	
CBDS	L.G.	90	60	2	8228	6467	TL	8771	8061	TL		8921	8488	TL	
CBDS	L.G.	90	60	3	8699	7704	TL	9752	8528	TL		9302	9254	TL	
CBDS	L.G.	90	60	4	8231	6160	TL	8806	7897	TL		8748	8635	TL	
CBDS	L.G.	90	60	5	7964	5807	TL	8702	7400	TL		8450	8096	TL	
CBDS	L.G.	90	60	6	8040	6426	TL	8862	7583	TL		8727	8493	TL	
CBDS	L.G.	90	60	7	8662	7803	TL	9368	7842	TL		9163	9141	TL	
CBDS	L.G.	90	60	8	7958	6664	TL	8468	7148	TL		8569	8280	TL	
CBDS	L.G.	90	60	9	8641	7443	TL	9713	8668	TL		9220	9166	TL	
CBDS	L.G.	90	75	0	6436	4591	TL	6901	4835	TL		6634	6333	TL	
CBDS	L.G.	90	75	1	6781	4786	TL	7685	5454	TL		7101	6837	TL	
CBDS	L.G.	90	75	2	6614	4734	TL	7299	4590	TL		7094	6639	TL	
CBDS	L.G.	90	75	3	6591	3357	TL	7336	5690	TL		7016	6704	TL	
CBDS	L.G.	90	75	4	7493	5500	TL	8142	6251	TL		7741	7571	TL	
CBDS	L.G.	90	75	5	6534	4642	TL	7299	5614	TL		6609	6517	TL	
CBDS	L.G.	90	75	6	6580	4836	TL	7438	5384	TL		7007	6693	TL	
CBDS	L.G.	90	75	7	6679	4119	TL	7076	5432	TL		6775	6481	TL	
CBDS	L.G.	90	75	8	6080	3222	TL	6947	4202	TL		6082	5914	TL	
CBDS	L.G.	90	75	9	6537	4769	TL	7458	4659	TL		6975	6631	TL	
CBDS	L.G.	100	30	0	16821	16821	78.6	**17765**	17765	1395.3	17801	20160	20160	2.3	
CBDS	L.G.	100	30	1	13953	13912	TL	**14741**	14734	TL	14758	16401	16401	676.4	
CBDS	L.G.	100	30	2	16569	16569	92.0	17452	17452	39.8	17452	20408	20408	2.4	
CBDS	L.G.	100	30	3	15622	15602	TL	**16528**	16528	2282.2	16530	18202	18202	229.5	

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Instance					Policy										
					optimal			largest gap				traversal			
					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	L.G.	100	30	4	14425	14420	TL	**15376**	15367	TL	15388	17385	17382	TL	
CBDS	L.G.	100	30	5	18248	18248	306.2	**19243**	19243	779.0	19356	21913	21913	18.6	
CBDS	L.G.	100	30	6	15803	15803	879.9	**16814**	16814	853.8	16901	19113	19113	82.8	
CBDS	L.G.	100	30	7	15626	15576	TL	**16441**	16424	TL	16452	18418	18418	299.6	
CBDS	L.G.	100	30	8	15115	15114	TL	**15882**	15873	TL	15950	17873	17873	50.5	
CBDS	L.G.	100	30	9	16417	16408	TL	**17309**	17307	TL	17430	19887	19887	51.3	
CBDS	L.G.	100	45	0	10661	9547	TL	**11438**	11074	TL	11539	11951	11893	TL	
CBDS	L.G.	100	45	1	10646	9945	TL	**11561**	10562	TL	11581	11821	11798	TL	
CBDS	L.G.	100	45	2	11371	9797	TL	12281	11764	TL	**12161**	12890	12876	TL	
CBDS	L.G.	100	45	3	11026	9408	TL	**11844**	11320	TL	11919	12064	12056	TL	
CBDS	L.G.	100	45	4	11330	9553	TL	**12146**	11033	TL	12180	12796	12783	TL	
CBDS	L.G.	100	45	5	10641	9326	TL	**11407**	11281	TL	11436	12094	12083	TL	
CBDS	L.G.	100	45	6	11730	10060	TL	**12633**	11094	TL	12718	12925	12921	TL	
CBDS	L.G.	100	45	7	11189	9498	TL	**11877**	11735	TL	11935	12479	12475	TL	
CBDS	L.G.	100	45	8	11434	9356	TL	**12222**	11316	TL	12280	12767	12674	TL	
CBDS	L.G.	100	45	9	11609	11163	TL	12402	11894	TL	**12390**	12953	12939	TL	
CBDS	L.G.	100	60	0	10104	8164	TL	10778	8517	TL	**10613**	10376	10354	TL	
CBDS	L.G.	100	60	1	9221	7170	TL	9984	8185	TL	**9832**	9868	9659	TL	
CBDS	L.G.	100	60	2	8865	6676	TL	9710	7303	TL	**9479**	9590	9293	TL	
CBDS	L.G.	100	60	3	9544	7143	TL	10608	8288	TL	**10132**	10201	9914	TL	
CBDS	L.G.	100	60	4	8829	7483	TL	10046	7783	TL	**9511**	9493	9213	TL	
CBDS	L.G.	100	60	5	8667	7256	TL	9405	8171	TL	**9184**	9061	8964	TL	
CBDS	L.G.	100	60	6	8257	5511	TL	8994	6930	TL	**8860**	8628	8497	TL	
CBDS	L.G.	100	60	7	9209	7234	TL	9963	7701	TL	**9464**	9457	9296	TL	
CBDS	L.G.	100	60	8	8223	6071	TL	9041	7618	TL	**8983**	9170	8823	TL	
CBDS	L.G.	100	60	9	8505	6333	TL	9384	7653	TL	**9152**	9194	8821	TL	
CBDS	L.G.	100	75	0	7301	4590	TL	8010	5885	TL	**7678**	7627	7281	TL	
CBDS	L.G.	100	75	1	7014	4238	TL	7864	5380	TL	**7468**	7081	6964	TL	
CBDS	L.G.	100	75	2	7468	4295	TL	8442	5719	TL	**7901**	7600	7477	TL	
CBDS	L.G.	100	75	3	7260	4629	TL	8008	4482	TL	**7596**	7380	7189	TL	
CBDS	L.G.	100	75	4	6789	3638	TL	7514	4535	TL	**7138**	7241	6824	TL	
CBDS	L.G.	100	75	5	8145	4634	TL	8780	4984	TL	**8445**	8296	7899	TL	
CBDS	L.G.	100	75	6	7204	4974	TL	7555	5613	TL	**7431**	7416	7215	TL	
CBDS	L.G.	100	75	7	7298	3839	TL	7894	4812	TL	**7631**	7360	7020	TL	
CBDS	L.G.	100	75	8	7417	4390	TL	8118	5440	TL	**7796**	7852	7494	TL	
CBDS	L.G.	100	75	9	7294	4637	TL	7996	5118	TL	**7609**	7373	7130	TL	
CBDS	S-S.	20	30	0	3743	3743	2.8	3940	3940	0.7		4612	4612	0.4	
CBDS	S-S.	20	30	1	3450	3450	9.0	3671	3671	2.2		4053	4053	0.7	
CBDS	S-S.	20	30	2	3207	3207	2.3	3440	3440	0.9		3885	3885	0.5	
CBDS	S-S.	20	30	3	3877	3877	6.9	4020	4020	1.0		4769	4769	0.4	
CBDS	S-S.	20	30	4	2922	2922	9.6	3103	3103	1.4		3465	3465	0.8	
CBDS	S-S.	20	30	5	3448	3448	9.3	3627	3627	1.1		4091	4091	0.3	
CBDS	S-S.	20	30	6	3959	3959	3.0	4069	4069	0.5		4755	4755	0.4	
CBDS	S-S.	20	30	7	3505	3505	7.3	3701	3701	1.5		4115	4115	0.5	
CBDS	S-S.	20	30	8	3052	3052	10.4	3230	3230	1.0		3592	3592	1.3	
CBDS	S-S.	20	30	9	2996	2996	74.0	3123	3123	7.1		3578	3578	3.6	
CBDS	S-S.	20	45	0	2686	2686	194.3	2957	2957	12.1		3134	3134	18.8	
CBDS	S-S.	20	45	1	2092	2092	206.4	2324	2324	95.7		2339	2339	34.5	
CBDS	S-S.	20	45	2	2621	2621	18.0	2810	2810	21.2		2996	2996	4.7	
CBDS	S-S.	20	45	3	2530	2530	840.9	2763	2763	29.3		2767	2767	6.8	
CBDS	S-S.	20	45	4	2583	2583	69.5	2796	2796	13.2		2932	2932	5.9	
CBDS	S-S.	20	45	5	2824	2824	21.9	2988	2988	2.6		3161	3161	1.1	
CBDS	S-S.	20	45	6	2326	2326	1188.5	2454	2454	481.4		2700	2700	262.2	
CBDS	S-S.	20	45	7	2064	2064	794.9	2213	2213	15.7		2296	2296	16.5	
CBDS	S-S.	20	45	8	2808	2808	58.1	3028	3028	30.8		3091	3091	8.1	
CBDS	S-S.	20	45	9	2343	2343	416.2	2580	2580	59.1		2587	2587	14.0	
CBDS	S-S.	20	60	0	2035	2035	19.3	2244	2244	3.9		2176	2176	1.3	
CBDS	S-S.	20	60	1	1835	1821	TL	2003	2003	1579.6		2053	2053	3079.1	
CBDS	S-S.	20	60	2	2335	2335	830.9	2524	2524	18.5		2510	2510	0.9	
CBDS	S-S.	20	60	3	1863	1863	1720.4	2049	2049	489.7		2038	2038	520.4	
CBDS	S-S.	20	60	4	1849	1831	TL	2042	2042	1002.9		1958	1958	753.3	
CBDS	S-S.	20	60	5	1697	1697	45.4	1844	1844	29.2		1925	1925	70.9	
CBDS	S-S.	20	60	6	1891	1890	TL	2102	2102	432.2		2058	2058	485.1	
CBDS	S-S.	20	60	7	1846	1793	TL	2053	2010	TL		1957	1929	TL	
CBDS	S-S.	20	60	8	1750	1741	TL	1818	1818	282.7		1947	1947	1056.9	
CBDS	S-S.	20	60	9	1902	1902	117.9	2063	2063	22.1		2074	2074	34.1	
CBDS	S-S.	20	75	0	1640	1640	601.7	1890	1890	43.9		1681	1681	14.8	
CBDS	S-S.	20	75	1	1584	1584	54.7	1767	1767	41.4		1696	1696	14.9	
CBDS	S-S.	20	75	2	1621	1621	809.6	1831	1831	40.3		1721	1721	32.5	
CBDS	S-S.	20	75	3	1521	1521	2836.3	1745	1745	131.9		1721	1700	TL	

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					Policy										
					optimal			largest gap				traversal			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	S-S.	20	75	4	1839	1836	TL	2016	2016	2167.7		2061	2061	410.1	
CBDS	S-S.	20	75	5	1573	1566	TL	1738	1738	430.0		1676	1676	1524.2	
CBDS	S-S.	20	75	6	1569	1549	TL	1676	1676	84.0		1691	1691	143.4	
CBDS	S-S.	20	75	7	1631	1631	631.8	1844	1844	131.3		1709	1709	32.0	
CBDS	S-S.	20	75	8	1622	1622	1652.5	1898	1898	264.6		1760	1760	431.6	
CBDS	S-S.	20	75	9	1517	1506	TL	1672	1672	250.4		1629	1629	214.4	
CBDS	S-S.	30	30	0	5782	5782	19.8	6068	6068	2.6		7174	7174	1.6	7174
CBDS	S-S.	30	30	1	5195	5195	47.5	5493	5493	9.7		6224	6224	3.2	6224
CBDS	S-S.	30	30	2	6996	6996	5.3	7479	7479	1.1		8534	8534	0.5	8534
CBDS	S-S.	30	30	3	4854	4854	5.2	5039	5039	1.5		5825	5825	0.5	5825
CBDS	S-S.	30	30	4	4789	4789	13.5	5039	5039	5.2		5696	5696	0.6	5696
CBDS	S-S.	30	30	5	4719	4719	5.4	4944	4944	3.0		5624	5624	8.0	5624
CBDS	S-S.	30	30	6	5023	5023	15.6	5279	5279	2.2		6043	6043	0.8	6043
CBDS	S-S.	30	30	7	5457	5457	7.4	5774	5774	1.3		6484	6484	0.6	6484
CBDS	S-S.	30	30	8	4255	4255	13.9	4430	4430	9.0		5162	5162	2.6	5162
CBDS	S-S.	30	30	9	5299	5299	22.1	5598	5598	11.2		6365	6365	1.2	6365
CBDS	S-S.	30	45	0	3246	3246	370.5	3513	3513	109.3		**3739**	3739	20.7	3856
CBDS	S-S.	30	45	1	4147	4147	224.9	4491	4491	49.3		**4701**	4701	20.7	4720
CBDS	S-S.	30	45	2	3579	3579	149.8	3838	3838	51.6		3920	3920	5.8	3920
CBDS	S-S.	30	45	3	3180	3180	553.1	3417	3417	543.8		**3646**	3646	275.8	3656
CBDS	S-S.	30	45	4	3866	3838	TL	4176	4175	TL		4386	4386	2186.0	4386
CBDS	S-S.	30	45	5	3659	3659	1164.2	3871	3871	34.1		4077	4077	579.0	4077
CBDS	S-S.	30	45	6	3834	3834	870.0	4123	4123	8.0		4409	4409	61.2	4409
CBDS	S-S.	30	45	7	3873	3869	TL	4104	4104	286.6		4486	4486	1540.9	4486
CBDS	S-S.	30	45	8	3920	3920	1286.1	4172	4172	82.2		4394	4394	1599.9	4394
CBDS	S-S.	30	45	9	3365	3362	TL	3584	3584	1677.1		**3842**	3842	47.3	3863
CBDS	S-S.	30	60	0	2902	2830	TL	3031	3008	TL		3180	3148	TL	3180
CBDS	S-S.	30	60	1	2916	2902	TL	3140	3140	352.4		**3223**	3223	1314.9	3267
CBDS	S-S.	30	60	2	2702	2695	TL	2991	2991	819.9		**2905**	2905	165.0	2921
CBDS	S-S.	30	60	3	2854	2770	TL	2973	2973	1007.8		**3016**	3016	450.8	3029
CBDS	S-S.	30	60	4	2916	2913	TL	3165	3165	174.4		**3157**	3157	243.7	3181
CBDS	S-S.	30	60	5	2612	2582	TL	2831	2825	TL		2895	2834	TL	2895
CBDS	S-S.	30	60	6	2964	2918	TL	3300	3225	TL		3180	3131	TL	**3168**
CBDS	S-S.	30	60	7	2545	2545	1072.6	2806	2806	195.4		**2800**	2800	147.4	2847
CBDS	S-S.	30	60	8	2854	2847	TL	3101	3101	771.1		**3069**	3069	78.1	3084
CBDS	S-S.	30	60	9	2465	2465	2527.0	2738	2738	479.8		2674	2674	601.5	2674
CBDS	S-S.	30	75	0	2208	2205	TL	2403	2403	222.9		2366	2366	215.9	2366
CBDS	S-S.	30	75	1	2272	2198	TL	2495	2495	3382.6		2457	2328	TL	**2409**
CBDS	S-S.	30	75	2	2760	2749	TL	3125	3125	2586.8		**2985**	2952	TL	3011
CBDS	S-S.	30	75	3	2113	2113	2382.5	2398	2398	189.3		**2311**	2311	2137.5	2315
CBDS	S-S.	30	75	4	2332	2232	TL	2501	2485	TL		2558	2493	TL	**2553**
CBDS	S-S.	30	75	5	2396	2384	TL	2630	2630	1069.6		**2532**	2532	19.0	2593
CBDS	S-S.	30	75	6	2294	2279	TL	2564	2555	TL		2573	2515	TL	2573
CBDS	S-S.	30	75	7	2368	2368	2955.1	2664	2664	591.5		**2547**	2547	349.5	2588
CBDS	S-S.	30	75	8	2319	2319	3454.6	2556	2556	285.5		**2525**	2525	191.9	2529
CBDS	S-S.	30	75	9	2326	2299	TL	2530	2525	TL		2591	2482	TL	**2521**
CBDS	S-S.	40	30	0	6721	6721	8.9	7077	7077	2.3		7986	7986	1.7	7986
CBDS	S-S.	40	30	1	5908	5908	104.7	6284	6284	44.6		6905	6905	1.5	6905
CBDS	S-S.	40	30	2	6780	6780	8.0	7118	7118	1.7		8253	8253	0.8	8253
CBDS	S-S.	40	30	3	8686	8686	11.0	9125	9125	2.0		10356	10356	0.6	10356
CBDS	S-S.	40	30	4	7141	7141	17.9	7587	7587	3.1		8608	8608	0.8	8608
CBDS	S-S.	40	30	5	6174	6174	10.2	6459	6459	12.3		7468	7468	0.7	7468
CBDS	S-S.	40	30	6	7395	7395	25.7	7889	7889	2.3		8788	8788	0.6	8788
CBDS	S-S.	40	30	7	6895	6895	57.9	7290	7290	9.6		8148	8148	1.7	8148
CBDS	S-S.	40	30	8	7136	7136	9.4	7504	7504	20.7		8596	8596	2.3	8596
CBDS	S-S.	40	30	9	6690	6690	188.0	7009	7009	5.0		8110	8110	6.1	8110
CBDS	S-S.	40	45	0	4553	4553	TL	4814	4814	140.9		5101	5101	82.7	5101
CBDS	S-S.	40	45	1	5229	5229	751.9	5668	5668	779.8		**5896**	5896	48.7	5903
CBDS	S-S.	40	45	2	5117	5117	1669.0	5493	5493	705.1		5634	5634	3.6	5634
CBDS	S-S.	40	45	3	4787	4787	1253.9	5185	5185	268.8		**5383**	5380	TL	5400
CBDS	S-S.	40	45	4	4678	4657	TL	5007	5007	314.9		5190	5169	TL	5190
CBDS	S-S.	40	45	5	4253	4249	TL	4531	4531	956.1		**4693**	4693	58.4	4707
CBDS	S-S.	40	45	6	4730	4717	TL	5073	5073	327.9		**5201**	5201	159.4	5233
CBDS	S-S.	40	45	7	4122	4122	1278.5	4448	4448	184.0		4623	4623	398.1	4623
CBDS	S-S.	40	45	8	4570	4570	2670.3	4827	4827	TL		5107	5101	TL	5107
CBDS	S-S.	40	45	9	4262	4182	TL	4546	4536	TL		**4853**	4853	646.5	4877
CBDS	S-S.	40	60	0	4241	4140	TL	4605	4549	TL		4543	4489	TL	4543
CBDS	S-S.	40	60	1	3595	3539	TL	3860	3857	TL		**4010**	3981	TL	4023
CBDS	S-S.	40	60	2	3764	3707	TL	4041	4038	TL		**4028**	3988	TL	4035
CBDS	S-S.	40	60	3	4186	3856	TL	4224	4224	2587.1		**4183**	4183	315.0	4263

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					Policy										
					optimal			largest gap				traversal			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	S-S.	40	60	4	4112	4111	TL	4515	4515	3009.4		**4575**	4575	475.9	4590
CBDS	S-S.	40	60	5	3690	3677	TL	4070	4041	TL		4094	4069	TL	4094
CBDS	S-S.	40	60	6	3546	3527	TL	3866	3866	1303.5		3830	3826	TL	3830
CBDS	S-S.	40	60	7	3292	3277	TL	3623	3623	TL		**3583**	3583	339.7	3634
CBDS	S-S.	40	60	8	3585	3573	TL	3890	3887	TL		**3869**	3856	TL	3887
CBDS	S-S.	40	60	9	3523	3523	969.7	3848	3848	TL		3951	3906	TL	**3943**
CBDS	S-S.	40	75	0	3122	3099	TL	3515	3500	TL		**3345**	3345	904.5	3354
CBDS	S-S.	40	75	1	2983	2909	TL	3301	3234	TL		3322	3233	TL	**3281**
CBDS	S-S.	40	75	2	3524	3432	TL	3902	3879	TL		3755	3683	TL	3755
CBDS	S-S.	40	75	3	3019	2905	TL	3306	3256	TL		3201	3126	TL	3201
CBDS	S-S.	40	75	4	3435	3132	TL	3509	3504	TL		**3343**	3343	314.8	3370
CBDS	S-S.	40	75	5	3022	2861	TL	3278	3160	TL		3292	3139	TL	**3257**
CBDS	S-S.	40	75	6	2943	2854	TL	3190	3190	1580.1		3187	3075	TL	**3122**
CBDS	S-S.	40	75	7	3355	3036	TL	3409	3402	TL		**3304**	3303	TL	3307
CBDS	S-S.	40	75	8	3010	2696	TL	3025	3015	TL		**2931**	2931	599.3	3024
CBDS	S-S.	40	75	9	2895	2858	TL	3162	3162	1147.4		3049	3048	TL	3049
CBDS	S-S.	50	30	0	7915	7915	77.3	8318	8318	27.8		9632	9632	3.5	9632
CBDS	S-S.	50	30	1	8800	8800	52.9	9447	9447	15.4		10445	10445	1.1	10445
CBDS	S-S.	50	30	2	6948	6948	857.3	7324	7324	51.0		8166	8166	68.0	8166
CBDS	S-S.	50	30	3	7847	7847	10.7	8111	8111	28.2		9379	9379	2.1	9379
CBDS	S-S.	50	30	4	7795	7795	227.8	8244	8244	103.5		8974	8974	22.5	8974
CBDS	S-S.	50	30	5	9085	9085	16.4	9626	9626	2.9		10748	10748	1.1	10748
CBDS	S-S.	50	30	6	8634	8634	36.2	9106	9106	4.5		10254	10254	0.9	10254
CBDS	S-S.	50	30	7	9685	9685	14.0	10127	10127	3.4		11516	11516	1.0	11516
CBDS	S-S.	50	30	8	8862	8862	134.6	9303	9303	13.4		10738	10738	7.2	10738
CBDS	S-S.	50	30	9	8842	8842	142.9	9394	9394	24.8		10548	10548	2.3	10548
CBDS	S-S.	50	45	0	5615	5615	2072.3	5999	5999	272.4		**6343**	6343	502.9	6399
CBDS	S-S.	50	45	1	5731	5724	TL	6252	6252	561.7		**6281**	6281	175.2	6302
CBDS	S-S.	50	45	2	6654	6646	TL	7168	7168	648.3		7309	7309	236.8	7309
CBDS	S-S.	50	45	3	6211	6209	TL	6679	6675	TL		**7015**	7015	125.9	7051
CBDS	S-S.	50	45	4	5548	5526	TL	5914	5902	TL		**6072**	6072	597.9	6076
CBDS	S-S.	50	45	5	6014	5998	TL	6405	6405	1850.1		**6748**	6748	614.2	6774
CBDS	S-S.	50	45	6	5934	5917	TL	6389	6389	1927.0		**6645**	6645	95.1	6788
CBDS	S-S.	50	45	7	6245	6221	TL	6648	6648	585.7		**6876**	6876	86.4	7072
CBDS	S-S.	50	45	8	5874	5862	TL	6368	6368	1434.2		**6399**	6399	154.1	6457
CBDS	S-S.	50	45	9	6300	6296	TL	6700	6700	2267.8		**7181**	7181	456.6	7221
CBDS	S-S.	50	60	0	4669	4646	TL	5010	5003	TL		**5125**	5125	874.4	5178
CBDS	S-S.	50	60	1	4605	4443	TL	4903	4901	TL		**4860**	4860	603.3	4963
CBDS	S-S.	50	60	2	4249	4167	TL	4548	4502	TL		**4601**	4563	TL	4628
CBDS	S-S.	50	60	3	4364	4289	TL	4733	4699	TL		**4707**	4662	TL	4745
CBDS	S-S.	50	60	4	5107	5016	TL	5560	5533	TL		**5638**	5605	TL	5661
CBDS	S-S.	50	60	5	4769	4704	TL	5137	5134	TL		**5172**	5162	TL	5222
CBDS	S-S.	50	60	6	5276	5203	TL	5728	5719	TL		**5562**	5531	TL	5612
CBDS	S-S.	50	60	7	4816	4549	TL	5032	4992	TL		**4940**	4940	1577.1	5014
CBDS	S-S.	50	60	8	4366	4319	TL	4737	4692	TL		4683	4681	TL	4683
CBDS	S-S.	50	60	9	4524	4513	TL	4919	4913	TL		**4928**	4926	TL	4979
CBDS	S-S.	50	75	0	3957	3738	TL	4258	4196	TL		**4031**	3995	TL	4072
CBDS	S-S.	50	75	1	3939	3861	TL	4381	4325	TL		**4126**	4091	TL	4139
CBDS	S-S.	50	75	2	3820	3726	TL	4245	4183	TL		**4085**	4022	TL	4146
CBDS	S-S.	50	75	3	3769	3663	TL	4211	4119	TL		**4017**	3957	TL	4022
CBDS	S-S.	50	75	4	3869	3558	TL	4052	4029	TL		3977	3888	TL	3977
CBDS	S-S.	50	75	5	3639	3518	TL	3925	3918	TL		3898	3815	TL	3898
CBDS	S-S.	50	75	6	3685	3557	TL	4076	4001	TL		**3822**	3784	TL	3849
CBDS	S-S.	50	75	7	3841	3685	TL	4276	4127	TL		4059	3898	TL	**4039**
CBDS	S-S.	50	75	8	4078	3781	TL	4244	4239	TL		**4061**	4057	TL	4136
CBDS	S-S.	50	75	9	4229	3807	TL	4619	4548	TL		**4500**	4421	TL	4509
CBDS	S-S.	60	30	0	10879	10879	55.6	11552	11552	261.5		13080	13080	4.5	13080
CBDS	S-S.	60	30	1	10501	10501	21.3	10950	10950	69.2		**12472**	12472	8.4	12557
CBDS	S-S.	60	30	2	9365	9365	402.3	9861	9861	186.3		**11264**	11264	15.7	11280
CBDS	S-S.	60	30	3	10143	10143	327.0	10638	10638	122.7		12014	12014	15.8	12014
CBDS	S-S.	60	30	4	10071	10071	41.1	10654	10654	65.1		11999	11999	3.4	11999
CBDS	S-S.	60	30	5	8955	8955	1233.3	9343	9343	251.0		**10631**	10631	14.6	10645
CBDS	S-S.	60	30	6	9224	9224	2156.7	9802	9802	1062.7		**10951**	10950	TL	10961
CBDS	S-S.	60	30	7	9712	9712	93.1	10127	10127	27.2		11914	11914	12.0	11914
CBDS	S-S.	60	30	8	10889	10889	250.7	11520	11520	61.8		**13486**	13486	4.2	13489
CBDS	S-S.	60	30	9	9187	9187	440.3	9713	9713	259.2		10964	10964	21.6	10964
CBDS	S-S.	60	45	0	7088	7085	TL	7517	7515	TL		**7840**	7840	312.4	7891
CBDS	S-S.	60	45	1	6486	6447	TL	6931	6899	TL		**7337**	7313	TL	7367
CBDS	S-S.	60	45	2	5436	5393	TL	5749	5741	TL		6089	6089	2282.4	6089
CBDS	S-S.	60	45	3	6116	6104	TL	6502	6502	1289.7		**6860**	6860	215.3	6922

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Instance					Policy										
					optimal			largest gap				traversal			
					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	S-S.	60	45	4	6525	6505	TL	7041	7040	TL		**7247**	7247	591.9	7313
CBDS	S-S.	60	45	5	6700	6658	TL	7157	7151	TL		**7516**	7516	851.9	7653
CBDS	S-S.	60	45	6	6992	6947	TL	7411	7404	TL		**7794**	7794	160.7	7961
CBDS	S-S.	60	45	7	6783	6775	TL	7352	7352	TL		**7595**	7595	657.2	7636
CBDS	S-S.	60	45	8	7571	7555	TL	8241	8238	TL		**8600**	8597	TL	8616
CBDS	S-S.	60	45	9	7068	7051	TL	7551	7551	1501.1		**8044**	8044	127.8	8084
CBDS	S-S.	60	60	0	5837	5117	TL	6364	6063	TL		**6099**	6099	1857.0	6184
CBDS	S-S.	60	60	1	5760	5372	TL	6275	5977	TL		**6013**	6013	2792.2	6081
CBDS	S-S.	60	60	2	5286	5144	TL	5823	5751	TL		**5729**	5684	TL	5753
CBDS	S-S.	60	60	3	5582	4824	TL	6104	5846	TL		**5699**	5699	869.1	5897
CBDS	S-S.	60	60	4	5599	5231	TL	6108	6062	TL		**5920**	5920	1174.5	5972
CBDS	S-S.	60	60	5	5270	4892	TL	5778	5504	TL		**5619**	5593	TL	5654
CBDS	S-S.	60	60	6	5005	4964	TL	5529	5447	TL		**5333**	5320	TL	5396
CBDS	S-S.	60	60	7	4951	4569	TL	5172	5152	TL		**5070**	5063	TL	5189
CBDS	S-S.	60	60	8	5543	5351	TL	6070	5859	TL		**5874**	5874	3239.7	5972
CBDS	S-S.	60	60	9	4925	4836	TL	5343	5250	TL		**5320**	5301	TL	5370
CBDS	S-S.	60	75	0	3968	3923	TL	4366	4313	TL		**4314**	4224	TL	4317
CBDS	S-S.	60	75	1	4964	4666	TL	5477	5068	TL		5371	5138	TL	**5288**
CBDS	S-S.	60	75	2	4452	4124	TL	4949	4794	TL		**4563**	4515	TL	4617
CBDS	S-S.	60	75	3	4478	3890	TL	4919	4823	TL		**4662**	4558	TL	4667
CBDS	S-S.	60	75	4	4420	3903	TL	4615	4518	TL		**4458**	4372	TL	4465
CBDS	S-S.	60	75	5	4321	3525	TL	4529	4465	TL		**4427**	4357	TL	4446
CBDS	S-S.	60	75	6	4168	3833	TL	4792	4550	TL		**4463**	4396	TL	4477
CBDS	S-S.	60	75	7	4429	3718	TL	4895	4758	TL		4801	4628	TL	**4766**
CBDS	S-S.	60	75	8	4786	3924	TL	5292	5037	TL		**4775**	4765	TL	4837
CBDS	S-S.	60	75	9	4671	4302	TL	4983	4815	TL		4846	4679	TL	**4828**
CBDS	S-S.	70	30	0	13818	13818	38.8	14434	14434	10.8		16922	16922	1.5	
CBDS	S-S.	70	30	1	11734	11734	95.6	12457	12457	26.2		14128	14128	10.6	
CBDS	S-S.	70	30	2	11641	11641	667.9	12335	12335	1069.6		13807	13807	153.5	
CBDS	S-S.	70	30	3	11961	11961	207.3	12472	12472	51.4		14453	14453	5.7	
CBDS	S-S.	70	30	4	11943	11943	37.9	12506	12506	101.4		14429	14429	1.5	
CBDS	S-S.	70	30	5	11769	11769	113.0	12410	12410	66.7		14347	14347	1.5	
CBDS	S-S.	70	30	6	11523	11523	1029.6	12196	12196	345.6		13695	13695	342.5	
CBDS	S-S.	70	30	7	11130	11130	2262.9	11669	11669	360.6		12962	12962	49.0	
CBDS	S-S.	70	30	8	12750	12750	348.1	13516	13516	13.4		15543	15543	13.2	
CBDS	S-S.	70	30	9	12637	12637	69.0	13459	13459	74.3		15130	15130	1.5	
CBDS	S-S.	70	45	0	6920	6755	TL	7378	7286	TL		7764	7762	TL	
CBDS	S-S.	70	45	1	8456	8387	TL	9182	9100	TL		9382	9382	1144.6	
CBDS	S-S.	70	45	2	8522	8479	TL	8991	8970	TL		9413	9405	TL	
CBDS	S-S.	70	45	3	7954	7411	TL	8499	8258	TL		8512	8512	3404.6	
CBDS	S-S.	70	45	4	8626	7683	TL	8826	8776	TL		9320	9294	TL	
CBDS	S-S.	70	45	5	7924	7775	TL	8377	8352	TL		8823	8823	422.9	
CBDS	S-S.	70	45	6	7974	7939	TL	8567	8512	TL		8968	8968	1124.5	
CBDS	S-S.	70	45	7	7788	7742	TL	8355	8039	TL		8695	8666	TL	
CBDS	S-S.	70	45	8	7641	7627	TL	8171	8143	TL		8535	8535	630.6	
CBDS	S-S.	70	45	9	8002	7982	TL	8551	8543	TL		8989	8972	TL	
CBDS	S-S.	70	60	0	5617	5390	TL	6118	6070	TL		6018	6010	TL	
CBDS	S-S.	70	60	1	6099	5840	TL	6977	6595	TL		6592	6537	TL	
CBDS	S-S.	70	60	2	6558	5406	TL	7019	6663	TL		6853	6821	TL	
CBDS	S-S.	70	60	3	5941	5664	TL	6255	6201	TL		6235	6215	TL	
CBDS	S-S.	70	60	4	6536	6281	TL	7079	7067	TL		7051	7036	TL	
CBDS	S-S.	70	60	5	6022	5106	TL	6297	6242	TL		6196	6155	TL	
CBDS	S-S.	70	60	6	6894	6340	TL	7474	7025	TL		7183	7176	TL	
CBDS	S-S.	70	60	7	6819	6249	TL	7798	6957	TL		7252	7249	TL	
CBDS	S-S.	70	60	8	6113	4962	TL	6554	6389	TL		6325	6294	TL	
CBDS	S-S.	70	60	9	6690	5769	TL	6979	6867	TL		7030	6961	TL	
CBDS	S-S.	70	75	0	5285	3959	TL	5843	4757	TL		5317	5313	TL	
CBDS	S-S.	70	75	1	5405	4199	TL	6257	5055	TL		5650	5453	TL	
CBDS	S-S.	70	75	2	5371	4055	TL	5931	5059	TL		5290	5247	TL	
CBDS	S-S.	70	75	3	5302	4047	TL	5820	4678	TL		5686	5329	TL	
CBDS	S-S.	70	75	4	5366	3534	TL	5670	4908	TL		5583	5392	TL	
CBDS	S-S.	70	75	5	5140	3408	TL	5740	4990	TL		5488	5168	TL	
CBDS	S-S.	70	75	6	5702	4572	TL	6251	5051	TL		6116	5707	TL	
CBDS	S-S.	70	75	7	5105	4032	TL	5398	4882	TL		5274	5149	TL	
CBDS	S-S.	70	75	8	4884	4248	TL	5136	4893	TL		5085	4874	TL	
CBDS	S-S.	70	75	9	5645	4407	TL	6279	5270	TL		6027	5671	TL	
CBDS	S-S.	80	30	0	14342	14342	28.4	14917	14917	81.9		17514	17514	2.0	
CBDS	S-S.	80	30	1	13152	13152	308.3	13956	13956	175.5		15709	15709	1.6	
CBDS	S-S.	80	30	2	12206	12206	1688.9	12792	12792	333.2		14711	14711	362.7	
CBDS	S-S.	80	30	3	12965	12965	993.5	13610	13610	190.9		15894	15894	15.7	

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					Policy										
					optimal			largest gap				traversal			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	S-S.	80	30	4	15182	15182	185.6	15927	15927	316.8		18323	18323	7.7	
CBDS	S-S.	80	30	5	13751	13751	174.4	14723	14723	157.7		16806	16806	9.9	
CBDS	S-S.	80	30	6	12667	12667	375.6	13457	13457	395.1		15017	15017	18.9	
CBDS	S-S.	80	30	7	12242	12242	1054.1	12903	12903	294.8		14373	14373	75.8	
CBDS	S-S.	80	30	8	12389	12389	581.5	12979	12979	1048.3		14190	14190	204.3	
CBDS	S-S.	80	30	9	11968	11960	*TL*	12608	12608	2113.2		14220	14220	235.1	
CBDS	S-S.	80	45	0	9994	9796	*TL*	10793	10761	*TL*		10934	10930	*TL*	
CBDS	S-S.	80	45	1	9317	8965	*TL*	9994	9941	*TL*		10193	10193	589.0	
CBDS	S-S.	80	45	2	9107	7716	*TL*	9396	9359	*TL*		10160	10157	*TL*	
CBDS	S-S.	80	45	3	8952	8552	*TL*	9702	9134	*TL*		9971	9944	*TL*	
CBDS	S-S.	80	45	4	8648	8571	*TL*	9271	9236	*TL*		9739	9724	*TL*	
CBDS	S-S.	80	45	5	8580	8136	*TL*	9158	9146	*TL*		9579	9567	*TL*	
CBDS	S-S.	80	45	6	9580	9546	*TL*	10181	10167	*TL*		10906	10891	*TL*	
CBDS	S-S.	80	45	7	9643	9233	*TL*	10251	10179	*TL*		10600	10596	*TL*	
CBDS	S-S.	80	45	8	8106	7966	*TL*	8649	8571	*TL*		8995	8986	*TL*	
CBDS	S-S.	80	45	9	9872	9460	*TL*	10540	10433	*TL*		11004	11004	1849.2	
CBDS	S-S.	80	60	0	7373	6093	*TL*	7994	7270	*TL*		7674	7638	*TL*	
CBDS	S-S.	80	60	1	6698	5803	*TL*	7462	7185	*TL*		7175	7157	*TL*	
CBDS	S-S.	80	60	2	7411	6255	*TL*	7920	7359	*TL*		7742	7697	*TL*	
CBDS	S-S.	80	60	3	7444	6145	*TL*	8138	7257	*TL*		7929	7849	*TL*	
CBDS	S-S.	80	60	4	6922	5598	*TL*	7439	6704	*TL*		7270	7224	*TL*	
CBDS	S-S.	80	60	5	6238	4956	*TL*	6699	6442	*TL*		6783	6523	*TL*	
CBDS	S-S.	80	60	6	8327	7312	*TL*	9057	7455	*TL*		8624	8601	*TL*	
CBDS	S-S.	80	60	7	7482	6218	*TL*	7781	7408	*TL*		7728	7712	*TL*	
CBDS	S-S.	80	60	8	7182	5574	*TL*	7784	6784	*TL*		7583	7564	*TL*	
CBDS	S-S.	80	60	9	7298	5890	*TL*	7571	7117	*TL*		7326	7321	*TL*	
CBDS	S-S.	80	75	0	6244	4162	*TL*	6851	5597	*TL*		6325	6281	*TL*	
CBDS	S-S.	80	75	1	6085	4108	*TL*	6570	5456	*TL*		6200	6007	*TL*	
CBDS	S-S.	80	75	2	5715	4028	*TL*	6326	4656	*TL*		6020	5864	*TL*	
CBDS	S-S.	80	75	3	6413	4991	*TL*	7501	6166	*TL*		6912	6646	*TL*	
CBDS	S-S.	80	75	4	5813	4496	*TL*	6403	5752	*TL*		5911	5847	*TL*	
CBDS	S-S.	80	75	5	6208	3855	*TL*	6954	5379	*TL*		6383	6233	*TL*	
CBDS	S-S.	80	75	6	5779	3190	*TL*	6214	4581	*TL*		5946	5625	*TL*	
CBDS	S-S.	80	75	7	5791	3885	*TL*	6437	5103	*TL*		5870	5701	*TL*	
CBDS	S-S.	80	75	8	6122	4916	*TL*	6871	6157	*TL*		6209	6129	*TL*	
CBDS	S-S.	80	75	9	5801	4265	*TL*	6393	4784	*TL*		5921	5852	*TL*	
CBDS	S-S.	90	30	0	14608	14608	642.7	15475	15475	1840.5		17124	17124	105.3	
CBDS	S-S.	90	30	1	14214	14214	781.7	15103	15103	1284.0		16967	16967	6.3	
CBDS	S-S.	90	30	2	14305	14305	1425.2	14989	14989	846.5		17031	17031	5.5	
CBDS	S-S.	90	30	3	14565	14565	428.0	15446	15446	402.7		17414	17414	4.0	
CBDS	S-S.	90	30	4	14256	14256	350.2	14970	14970	1455.5		17180	17180	44.0	
CBDS	S-S.	90	30	5	14411	14411	1196.1	15298	15276	*TL*		17224	17210	*TL*	
CBDS	S-S.	90	30	6	14464	14464	1046.3	15157	15157	1303.4		16896	16896	68.6	
CBDS	S-S.	90	30	7	15903	15903	60.5	16870	16870	8.5		19064	19064	2.4	
CBDS	S-S.	90	30	8	15054	15054	1398.9	15826	15824	*TL*		17842	17842	96.8	
CBDS	S-S.	90	30	9	13581	13581	*TL*	14256	14256	2311.7		16160	16160	29.7	
CBDS	S-S.	90	45	0	10042	8342	*TL*	10692	10618	*TL*		11260	11197	*TL*	
CBDS	S-S.	90	45	1	10775	9575	*TL*	11692	10820	*TL*		11998	11997	*TL*	
CBDS	S-S.	90	45	2	9806	8931	*TL*	10310	9925	*TL*		10844	10827	*TL*	
CBDS	S-S.	90	45	3	10306	9911	*TL*	11081	11069	*TL*		11444	11435	*TL*	
CBDS	S-S.	90	45	4	9381	7669	*TL*	10059	9864	*TL*		10376	10371	*TL*	
CBDS	S-S.	90	45	5	10094	9520	*TL*	10674	10643	*TL*		11263	11257	*TL*	
CBDS	S-S.	90	45	6	10559	9720	*TL*	11326	10844	*TL*		11624	11624	841.3	
CBDS	S-S.	90	45	7	10212	8803	*TL*	10982	10629	*TL*		11380	11373	*TL*	
CBDS	S-S.	90	45	8	10499	9068	*TL*	10647	10596	*TL*		11011	11011	855.9	
CBDS	S-S.	90	45	9	10069	9494	*TL*	10778	10632	*TL*		11194	11191	*TL*	
CBDS	S-S.	90	60	0	8333	7411	*TL*	9163	7501	*TL*		8890	8754	*TL*	
CBDS	S-S.	90	60	1	8335	6645	*TL*	9390	7514	*TL*		8692	8644	*TL*	
CBDS	S-S.	90	60	2	8384	6034	*TL*	8998	7524	*TL*		8528	8495	*TL*	
CBDS	S-S.	90	60	3	8254	7173	*TL*	8939	7295	*TL*		8823	8759	*TL*	
CBDS	S-S.	90	60	4	7429	5196	*TL*	8092	7294	*TL*		8105	7762	*TL*	
CBDS	S-S.	90	60	5	8762	6146	*TL*	9542	7464	*TL*		8711	8679	*TL*	
CBDS	S-S.	90	60	6	7834	5870	*TL*	8600	7664	*TL*		8550	8182	*TL*	
CBDS	S-S.	90	60	7	8391	6728	*TL*	8923	7764	*TL*		8574	8493	*TL*	
CBDS	S-S.	90	60	8	7974	6125	*TL*	8676	7410	*TL*		8409	8373	*TL*	
CBDS	S-S.	90	60	9	7847	6565	*TL*	8475	8271	*TL*		8361	8312	*TL*	
CBDS	S-S.	90	75	0	6457	3484	*TL*	7011	5398	*TL*		6574	6222	*TL*	
CBDS	S-S.	90	75	1	6504	3660	*TL*	7318	5324	*TL*		6866	6467	*TL*	
CBDS	S-S.	90	75	2	6978	5356	*TL*	7738	5623	*TL*		7467	7142	*TL*	
CBDS	S-S.	90	75	3	6509	4676	*TL*	7146	5025	*TL*		6645	6555	*TL*	

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					Policy										
					optimal			largest gap				traversal			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\lvert O \rvert$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
CBDS	S-S.	90	75	4	6706	4526	TL	7415	4258	TL		7047	6674	TL	
CBDS	S-S.	90	75	5	6392	4194	TL	7133	5615	TL		6609	6542	TL	
CBDS	S-S.	90	75	6	6960	5283	TL	7796	5589	TL		6923	6842	TL	
CBDS	S-S.	90	75	7	7345	5028	TL	8134	5955	TL		7480	7430	TL	
CBDS	S-S.	90	75	8	6698	4757	TL	7493	6003	TL		6865	6664	TL	
CBDS	S-S.	90	75	9	6756	3970	TL	7800	4949	TL		7128	6725	TL	
CBDS	S-S.	100	30	0	15039	15033	TL	15858	15858	TL		18013	18013	1113.5	
CBDS	S-S.	100	30	1	18165	18165	450.7	19166	19166	145.0		21701	21701	17.2	
CBDS	S-S.	100	30	2	16421	16421	3309.6	17312	17312	1130.8		19143	19143	36.9	
CBDS	S-S.	100	30	3	15753	15753	1404.3	16558	16558	887.5		18650	18650	77.2	
CBDS	S-S.	100	30	4	18571	18571	88.2	19671	19671	509.6		22278	22278	2.7	
CBDS	S-S.	100	30	5	16620	16560	TL	17458	17458	860.1		19510	19510	95.8	
CBDS	S-S.	100	30	6	16140	16140	727.8	16961	16961	505.7		19123	19123	42.9	
CBDS	S-S.	100	30	7	16315	16315	393.5	17191	17191	415.2		19574	19574	68.7	
CBDS	S-S.	100	30	8	16357	16357	1379.0	17117	17117	447.3		19279	19273	TL	
CBDS	S-S.	100	30	9	14914	14914	1297.1	15703	15703	2171.3		17531	17531	199.6	
CBDS	S-S.	100	45	0	10172	9164	TL	10792	10550	TL		11437	11427	TL	
CBDS	S-S.	100	45	1	11591	10497	TL	12430	11772	TL		12916	12903	TL	
CBDS	S-S.	100	45	2	11297	9740	TL	12002	11259	TL		12571	12550	TL	
CBDS	S-S.	100	45	3	11847	9985	TL	12742	12601	TL		13250	13241	TL	
CBDS	S-S.	100	45	4	10761	10163	TL	11459	11046	TL		11917	11902	TL	
CBDS	S-S.	100	45	5	10720	10088	TL	11377	10614	TL		11943	11884	TL	
CBDS	S-S.	100	45	6	11165	9744	TL	11943	11884	TL		12275	12275	2020.0	
CBDS	S-S.	100	45	7	10597	9681	TL	11134	10172	TL		11512	11493	TL	
CBDS	S-S.	100	45	8	11155	9175	TL	11928	11602	TL		12169	12142	TL	
CBDS	S-S.	100	45	9	10343	9052	TL	11031	10462	TL		11756	11699	TL	
CBDS	S-S.	100	60	0	8445	6228	TL	9189	7714	TL		9073	8828	TL	
CBDS	S-S.	100	60	1	8946	6800	TL	9876	8241	TL		9635	9582	TL	
CBDS	S-S.	100	60	2	8376	6045	TL	9022	7052	TL		8972	8865	TL	
CBDS	S-S.	100	60	3	8093	5920	TL	8669	7338	TL		8755	8460	TL	
CBDS	S-S.	100	60	4	9029	7601	TL	10245	7316	TL		9411	9374	TL	
CBDS	S-S.	100	60	5	8825	6119	TL	9623	7752	TL		9314	9199	TL	
CBDS	S-S.	100	60	6	8794	6835	TL	9321	8279	TL		9277	9101	TL	
CBDS	S-S.	100	60	7	8932	6634	TL	9845	7496	TL		9745	9485	TL	
CBDS	S-S.	100	60	8	9967	6717	TL	10065	8726	TL		10239	9858	TL	
CBDS	S-S.	100	60	9	8147	6227	TL	9435	8054	TL		8773	8605	TL	
CBDS	S-S.	100	75	0	7469	4486	TL	8292	4589	TL		7648	7425	TL	
CBDS	S-S.	100	75	1	7888	5607	TL	8689	5745	TL		8444	8074	TL	
CBDS	S-S.	100	75	2	7340	4029	TL	8473	5464	TL		7827	7495	TL	
CBDS	S-S.	100	75	3	8197	5263	TL	8789	6004	TL		8535	8090	TL	
CBDS	S-S.	100	75	4	7243	4476	TL	8303	4821	TL		7773	7390	TL	
CBDS	S-S.	100	75	5	8066	5477	TL	8937	6783	TL		8527	8122	TL	
CBDS	S-S.	100	75	6	7367	5310	TL	8151	5538	TL		7654	7412	TL	
CBDS	S-S.	100	75	7	7033	4360	TL	7594	4841	TL		7447	7047	TL	
CBDS	S-S.	100	75	8	7003	4877	TL	7993	5597	TL		7526	7299	TL	
CBDS	S-S.	100	75	9	8146	5746	TL	8827	5849	TL		8795	8403	TL	
UDS	L.G.	20	30	0	4316	4316	19.7	4719	4719	12.5		5270	5270	19.4	
UDS	L.G.	20	30	1	4531	4531	10.1	4943	4943	6.1		5384	5384	3.3	
UDS	L.G.	20	30	2	4982	4982	12.7	5450	5450	1.0		6093	6093	0.6	
UDS	L.G.	20	30	3	3671	3671	278.0	4006	4006	5.0		4201	4201	2.4	
UDS	L.G.	20	30	4	3464	3464	24.2	3731	3731	29.5		4023	4023	1.3	
UDS	L.G.	20	30	5	4511	4511	22.5	4920	4920	1.0		5122	5122	0.7	
UDS	L.G.	20	30	6	4541	4541	23.5	4973	4973	13.3		5360	5360	3.2	
UDS	L.G.	20	30	7	5157	5157	30.0	5585	5585	2.9		6001	6001	1.9	
UDS	L.G.	20	30	8	5582	5582	24.7	6029	6029	6.7		6650	6650	38.9	
UDS	L.G.	20	30	9	5395	5395	6.8	5880	5880	3.1		6525	6525	1.1	
UDS	L.G.	20	45	0	3328	3328	3518.1	3677	3677	145.9		3670	3670	69.3	
UDS	L.G.	20	45	1	3241	3241	3058.0	3686	3686	72.2		3523	3523	91.3	
UDS	L.G.	20	45	2	3230	3213	TL	3603	3603	271.9		3551	3551	375.5	
UDS	L.G.	20	45	3	2930	2930	166.6	3354	3354	32.9		3236	3236	3.9	
UDS	L.G.	20	45	4	3202	3202	170.2	3643	3643	698.5		3580	3580	18.8	
UDS	L.G.	20	45	5	3258	3258	804.9	3668	3668	28.8		3579	3579	92.9	
UDS	L.G.	20	45	6	3331	3317	TL	3678	3678	448.9		3629	3629	970.2	
UDS	L.G.	20	45	7	2896	2896	25.4	3277	3277	13.6		3166	3166	5.2	
UDS	L.G.	20	45	8	3454	3454	51.3	3939	3939	16.4		3749	3749	15.0	
UDS	L.G.	20	45	9	3453	3453	78.1	3818	3818	30.9		3754	3754	1.9	
UDS	L.G.	20	60	0	2767	2767	414.0	3146	3146	22.2		3045	3045	30.2	
UDS	L.G.	20	60	1	2507	2452	TL	2907	2907	2585.2		2616	2600	TL	
UDS	L.G.	20	60	2	2514	2514	1251.8	2900	2900	70.2		2683	2683	568.5	
UDS	L.G.	20	60	3	2308	2284	TL	2698	2697	TL		2390	2390	69.9	

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					Policy										
					optimal			largest gap				traversal			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
UDS	L.G.	20	60	4	2401	2364	*TL*	2733	2723	*TL*		2679	2571	*TL*	
UDS	L.G.	20	60	5	2494	2494	*TL*	2854	2854	363.4		2742	2708	*TL*	
UDS	L.G.	20	60	6	2327	2303	*TL*	2705	2705	273.0		2483	2483	123.2	
UDS	L.G.	20	60	7	2514	2480	*TL*	2922	2922	3210.1		2703	2680	*TL*	
UDS	L.G.	20	60	8	2558	2558	2015.1	2984	2984	59.2		2750	2750	462.8	
UDS	L.G.	20	60	9	2871	2839	*TL*	3339	3339	2448.1		3135	3113	*TL*	
UDS	L.G.	20	75	0	2045	2026	*TL*	2434	2434	2927.7		2137	2137	88.8	
UDS	L.G.	20	75	1	2579	2575	*TL*	3065	3065	1514.0		2748	2712	*TL*	
UDS	L.G.	20	75	2	2130	2130	449.4	2612	2612	508.0		2201	2201	86.0	
UDS	L.G.	20	75	3	1998	1902	*TL*	2382	2308	*TL*		2200	2038	*TL*	
UDS	L.G.	20	75	4	1580	1553	*TL*	1845	1845	1364.8		1665	1635	*TL*	
UDS	L.G.	20	75	5	2294	2294	333.5	2752	2752	107.5		2461	2461	85.9	
UDS	L.G.	20	75	6	1916	1842	*TL*	2205	2205	2275.2		2188	1984	*TL*	
UDS	L.G.	20	75	7	2558	2406	*TL*	2888	2868	*TL*		2755	2559	*TL*	
UDS	L.G.	20	75	8	1896	1842	*TL*	2233	2177	*TL*		2196	1955	*TL*	
UDS	L.G.	20	75	9	1984	1910	*TL*	2359	2276	*TL*		2150	1991	*TL*	
UDS	L.G.	30	30	0	6050	6050	54.5	6748	6748	16.0		7391	7391	18.1	
UDS	L.G.	30	30	1	6122	6122	102.5	6559	6559	12.6		7249	7249	1.1	
UDS	L.G.	30	30	2	6712	6712	41.1	7317	7317	3.1		7776	7776	1.3	
UDS	L.G.	30	30	3	7283	7283	11.2	7831	7831	2.7		8783	8783	0.5	
UDS	L.G.	30	30	4	8595	8595	14.7	9415	9415	4.0		10329	10329	0.6	
UDS	L.G.	30	30	5	8180	8180	25.9	8735	8735	3.6		9781	9781	0.6	
UDS	L.G.	30	30	6	6167	6167	147.3	6772	6772	13.9		7237	7237	3.7	
UDS	L.G.	30	30	7	7480	7480	38.7	8293	8293	12.5		8833	8833	1.6	
UDS	L.G.	30	30	8	6533	6533	578.4	7089	7089	17.7		7637	7637	27.8	
UDS	L.G.	30	30	9	6050	6050	300.4	6628	6628	39.6		6931	6931	418.5	
UDS	L.G.	30	45	0	5140	5117	*TL*	5819	5815	*TL*		5576	5576	262.8	
UDS	L.G.	30	45	1	5027	5026	*TL*	5752	5752	2798.3		5523	5523	409.8	
UDS	L.G.	30	45	2	4722	4701	*TL*	5278	5271	*TL*		5156	5156	1923.5	
UDS	L.G.	30	45	3	4919	4919	1976.5	5575	5575	283.3		5312	5312	87.7	
UDS	L.G.	30	45	4	5177	5129	*TL*	5859	5853	*TL*		5754	5717	*TL*	
UDS	L.G.	30	45	5	5185	5157	*TL*	5828	5820	*TL*		5662	5662	26.8	
UDS	L.G.	30	45	6	4894	4894	1411.5	5497	5497	434.7		5413	5413	22.1	
UDS	L.G.	30	45	7	4488	4472	*TL*	5043	5043	1181.2		5043	5028	*TL*	
UDS	L.G.	30	45	8	4884	4884	1171.2	5542	5542	299.8		5444	5444	33.6	
UDS	L.G.	30	45	9	5379	5379	2198.2	6110	6110	315.9		5851	5851	36.8	
UDS	L.G.	30	60	0	3948	3863	*TL*	4549	4507	*TL*		4267	4227	*TL*	
UDS	L.G.	30	60	1	4045	3850	*TL*	4634	4505	*TL*		4296	4169	*TL*	
UDS	L.G.	30	60	2	3528	3501	*TL*	4036	4033	*TL*		3734	3734	65.2	
UDS	L.G.	30	60	3	3452	3452	3322.4	4054	4054	1577.6		3714	3714	318.4	
UDS	L.G.	30	60	4	3972	3954	*TL*	4640	4631	*TL*		4236	4236	23.1	
UDS	L.G.	30	60	5	4458	4311	*TL*	5085	4969	*TL*		4661	4602	*TL*	
UDS	L.G.	30	60	6	3481	3441	*TL*	4007	3985	*TL*		3656	3656	2068.2	
UDS	L.G.	30	60	7	3884	3803	*TL*	4413	4380	*TL*		4216	4192	*TL*	
UDS	L.G.	30	60	8	3898	3807	*TL*	4530	4412	*TL*		4256	4099	*TL*	
UDS	L.G.	30	60	9	3509	3507	*TL*	4043	4043	904.6		3756	3756	14.6	
UDS	L.G.	30	75	0	3089	2959	*TL*	3714	3574	*TL*		3278	3155	*TL*	
UDS	L.G.	30	75	1	2579	2514	*TL*	3050	3032	*TL*		2655	2649	*TL*	
UDS	L.G.	30	75	2	3055	3016	*TL*	3713	3694	*TL*		3152	3152	282.8	
UDS	L.G.	30	75	3	3062	2978	*TL*	3601	3572	*TL*		3154	3153	*TL*	
UDS	L.G.	30	75	4	3475	3052	*TL*	3711	3692	*TL*		3284	3284	103.7	
UDS	L.G.	30	75	5	3254	3254	*TL*	4032	3869	*TL*		3487	3478	*TL*	
UDS	L.G.	30	75	6	3033	3003	*TL*	3614	3602	*TL*		3223	3207	*TL*	
UDS	L.G.	30	75	7	3050	2844	*TL*	3538	3388	*TL*		3190	2999	*TL*	
UDS	L.G.	30	75	8	3000	2805	*TL*	3514	3372	*TL*		3224	2984	*TL*	
UDS	L.G.	30	75	9	2985	2777	*TL*	3389	3268	*TL*		3213	3025	*TL*	
UDS	L.G.	40	30	0	9690	9690	341.0	10539	10539	120.0	10539	11153	11153	39.5	
UDS	L.G.	40	30	1	9030	9030	88.3	9734	9734	18.1	9734	10802	10802	1.3	
UDS	L.G.	40	30	2	8152	8152	679.2	**8810**	8810	168.2	8918	9396	9396	2.9	
UDS	L.G.	40	30	3	8362	8362	609.3	8989	8989	2024.7	8989	9766	9766	3.2	
UDS	L.G.	40	30	4	8519	8519	123.2	9256	9256	29.9	9256	10115	10115	1.4	
UDS	L.G.	40	30	5	9194	9194	91.1	10048	10048	343.6	10048	10826	10826	0.7	
UDS	L.G.	40	30	6	8238	8238	160.6	9049	9049	260.2	9049	9774	9774	2.8	
UDS	L.G.	40	30	7	7865	7865	139.0	8548	8548	109.7	8548	9171	9171	28.0	
UDS	L.G.	40	30	8	9394	9394	84.6	10263	10263	43.4	10263	11182	11182	1.0	
UDS	L.G.	40	30	9	8673	8673	567.9	9530	9530	1733.5	9530	10172	10172	12.0	
UDS	L.G.	40	45	0	7109	7086	*TL*	8028	8006	*TL*	8028	7876	7855	*TL*	
UDS	L.G.	40	45	1	6688	6668	*TL*	7441	7441	1304.5	7441	7400	7400	21.0	
UDS	L.G.	40	45	2	6105	6101	*TL*	6881	6874	*TL*	6881	6737	6723	*TL*	
UDS	L.G.	40	45	3	5976	5960	*TL*	6834	6822	*TL*	6834	6491	6491	146.7	

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Instance					Policy										
					optimal			largest gap				traversal			
					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
UDS	L.G.	40	45	4	6473	6231	TL	7314	7032	TL	**7170**	6866	6866	81.8	
UDS	L.G.	40	45	5	5709	5683	TL	**6345**	6327	TL	6373	6251	6250	TL	
UDS	L.G.	40	45	6	6586	6583	TL	7245	7245	2389.8	7245	7221	7221	220.5	
UDS	L.G.	40	45	7	6172	6163	TL	**6964**	6933	TL	6965	6697	6697	95.1	
UDS	L.G.	40	45	8	7004	6923	TL	**7903**	7903	2952.8	7911	7819	7819	53.2	
UDS	L.G.	40	45	9	5661	5652	TL	**6363**	6356	TL	6397	6156	6156	464.0	
UDS	L.G.	40	60	0	5366	4848	TL	5614	5562	TL	**5591**	5299	5248	TL	
UDS	L.G.	40	60	1	4346	3816	TL	5046	4884	TL	**4977**	4571	4519	TL	
UDS	L.G.	40	60	2	4841	4569	TL	5622	5397	TL	**5496**	5261	5097	TL	
UDS	L.G.	40	60	3	5382	4953	TL	6325	5709	TL	**5826**	5206	5206	82.8	
UDS	L.G.	40	60	4	5307	4920	TL	6110	6036	TL	**6082**	5547	5547	529.0	
UDS	L.G.	40	60	5	4940	4806	TL	5649	5435	TL	**5593**	5304	5161	TL	
UDS	L.G.	40	60	6	4852	4576	TL	5693	5145	TL	**5595**	5186	5133	TL	
UDS	L.G.	40	60	7	5473	5190	TL	6321	6029	TL	**6256**	5753	5708	TL	
UDS	L.G.	40	60	8	4802	4473	TL	5541	5436	TL	**5509**	5122	5075	TL	
UDS	L.G.	40	60	9	4884	4884	TL	**5808**	5778	TL	5861	5270	5270	275.4	
UDS	L.G.	40	75	0	4437	3598	TL	5297	4185	TL	**4780**	4236	4204	TL	
UDS	L.G.	40	75	1	4087	3843	TL	4773	4502	TL	**4746**	4294	4241	TL	
UDS	L.G.	40	75	2	4536	3579	TL	4896	4415	TL	**4848**	4240	4216	TL	
UDS	L.G.	40	75	3	3965	3514	TL	4915	4433	TL	**4495**	4208	3994	TL	
UDS	L.G.	40	75	4	4020	3655	TL	4933	4033	TL	**4584**	4248	4202	TL	
UDS	L.G.	40	75	5	4600	3912	TL	5394	4924	TL	**5251**	4839	4648	TL	
UDS	L.G.	40	75	6	3441	3239	TL	4161	3853	TL	**4044**	3559	3467	TL	
UDS	L.G.	40	75	7	4064	3511	TL	4786	4549	TL	**4719**	4318	4249	TL	
UDS	L.G.	40	75	8	4580	4263	TL	5403	4711	TL	**5130**	4662	4654	TL	
UDS	L.G.	40	75	9	4211	3614	TL	5278	4071	TL	**4666**	4148	4069	TL	
UDS	L.G.	50	30	0	12688	12688	31.2	13859	13859	107.7		15000	15000	1.3	
UDS	L.G.	50	30	1	10383	10383	698.5	11342	11342	282.1		12011	12011	3.3	
UDS	L.G.	50	30	2	12547	12547	110.5	13442	13442	489.3		14611	14611	2.4	
UDS	L.G.	50	30	3	10757	10757	726.8	11575	11575	2000.0		12472	12472	15.9	
UDS	L.G.	50	30	4	10021	10019	TL	10971	10971	2827.8		11649	11649	14.6	
UDS	L.G.	50	30	5	13152	13152	254.9	14143	14143	1694.9		15374	15374	3.2	
UDS	L.G.	50	30	6	10501	10501	696.7	11480	11480	1962.8		12582	12582	9.4	
UDS	L.G.	50	30	7	12860	12860	255.2	13860	13860	146.2		15334	15334	1.0	
UDS	L.G.	50	30	8	12262	12262	264.0	13415	13415	696.1		14104	14104	2.7	
UDS	L.G.	50	30	9	11052	11052	196.7	12206	12206	118.6		12941	12941	1.4	
UDS	L.G.	50	45	0	7860	7103	TL	8797	8446	TL		8549	8549	1752.3	
UDS	L.G.	50	45	1	6997	6518	TL	8249	7625	TL		7588	7580	TL	
UDS	L.G.	50	45	2	6996	6796	TL	8201	7772	TL		7620	7620	2726.3	
UDS	L.G.	50	45	3	6985	6539	TL	7873	7839	TL		7699	7699	473.7	
UDS	L.G.	50	45	4	7341	6728	TL	8287	8132	TL		8047	8043	TL	
UDS	L.G.	50	45	5	8467	8108	TL	9452	9080	TL		9015	9015	1529.7	
UDS	L.G.	50	45	6	7916	7827	TL	8847	8815	TL		8658	8655	TL	
UDS	L.G.	50	45	7	7237	6609	TL	7755	7701	TL		7540	7506	TL	
UDS	L.G.	50	45	8	7462	7087	TL	8379	8335	TL		8285	8208	TL	
UDS	L.G.	50	45	9	7746	7067	TL	8981	8132	TL		8406	8406	602.8	
UDS	L.G.	50	60	0	7010	5956	TL	8049	6788	TL		7029	7029	266.4	
UDS	L.G.	50	60	1	5791	4670	TL	6825	5666	TL		6107	6089	TL	
UDS	L.G.	50	60	2	5636	4032	TL	6705	4919	TL		5699	5696	TL	
UDS	L.G.	50	60	3	5728	4884	TL	6547	5665	TL		6054	6027	TL	
UDS	L.G.	50	60	4	6267	5025	TL	7216	6108	TL		6685	6527	TL	
UDS	L.G.	50	60	5	5674	4968	TL	6488	5949	TL		6189	6015	TL	
UDS	L.G.	50	60	6	5792	4220	TL	6680	4938	TL		5756	5756	1032.5	
UDS	L.G.	50	60	7	5776	5222	TL	6597	5336	TL		6109	6038	TL	
UDS	L.G.	50	60	8	6309	5089	TL	7222	5955	TL		6660	6651	TL	
UDS	L.G.	50	60	9	6306	5547	TL	7290	5682	TL		6384	6366	TL	
UDS	L.G.	50	75	0	4983	3454	TL	6445	3976	TL		5305	5134	TL	
UDS	L.G.	50	75	1	5245	4280	TL	6577	4094	TL		5383	5368	TL	
UDS	L.G.	50	75	2	4949	4003	TL	6017	4313	TL		5155	4988	TL	
UDS	L.G.	50	75	3	4968	3675	TL	5923	2597	TL		5155	5088	TL	
UDS	L.G.	50	75	4	4960	3777	TL	6434	4536	TL		5155	4976	TL	
UDS	L.G.	50	75	5	4479	2721	TL	5236	3096	TL		4690	4577	TL	
UDS	L.G.	50	75	6	5604	4294	TL	6582	4190	TL		5735	5565	TL	
UDS	L.G.	50	75	7	5001	4005	TL	5979	4247	TL		5213	5162	TL	
UDS	L.G.	50	75	8	5624	3808	TL	6629	4263	TL		5423	5422	TL	
UDS	L.G.	50	75	9	5554	4283	TL	6557	4641	TL		5796	5707	TL	
UDS	L.G.	60	30	0	13457	13457	78.6	**14788**	14788	116.2	14795	15843	15843	3.7	
UDS	L.G.	60	30	1	14052	14052	379.9	15506	15506	919.4	15506	16427	16427	10.7	
UDS	L.G.	60	30	2	15291	15291	556.6	16582	16579	TL	16582	18622	18616	TL	
UDS	L.G.	60	30	3	12760	12760	698.9	**14073**	14073	2637.9	14078	14790	14790	13.8	

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					Policy										
					optimal			largest gap				traversal			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
UDS	L.G.	60	30	4	14799	14799	266.4	16017	16017	2069.7	16017	17832	17832	13.1	
UDS	L.G.	60	30	5	12032	12032	1660.7	**13024**	13024	1295.2	13028	14184	14184	219.7	
UDS	L.G.	60	30	6	15252	15252	59.5	**16498**	16498	445.6	16564	18327	18327	1.3	
UDS	L.G.	60	30	7	12501	12501	2363.1	13785	13770	*TL*	13785	14694	14694	17.4	
UDS	L.G.	60	30	8	12396	12393	*TL*	13587	13554	*TL*	13587	14784	14770	*TL*	
UDS	L.G.	60	30	9	15080	15080	246.1	16186	16186	505.9	16186	17719	17719	1.5	
UDS	L.G.	60	45	0	9943	8977	*TL*	**11307**	10829	*TL*	11384	10933	10933	1622.2	
UDS	L.G.	60	45	1	8828	7733	*TL*	**9936**	9494	*TL*	9973	9729	9729	344.2	
UDS	L.G.	60	45	2	8949	7967	*TL*	**10033**	8875	*TL*	10064	9742	9731	*TL*	
UDS	L.G.	60	45	3	8220	7886	*TL*	9507	8501	*TL*	**9166**	8936	8936	2545.6	
UDS	L.G.	60	45	4	8736	7531	*TL*	10223	8965	*TL*	**10039**	9648	9648	532.4	
UDS	L.G.	60	45	5	8738	7425	*TL*	9734	8841	*TL*	**9727**	9484	9465	*TL*	
UDS	L.G.	60	45	6	9213	8332	*TL*	10294	9254	*TL*	**10247**	10277	10277	1328.5	
UDS	L.G.	60	45	7	8726	7663	*TL*	9697	8809	*TL*	**9558**	9466	9466	546.5	
UDS	L.G.	60	45	8	9422	8507	*TL*	10696	8885	*TL*	**10661**	10255	10255	559.8	
UDS	L.G.	60	45	9	9333	8821	*TL*	10535	9393	*TL*	**10517**	10331	10310	*TL*	
UDS	L.G.	60	60	0	7216	5675	*TL*	8460	6408	*TL*	**8339**	7656	7596	*TL*	
UDS	L.G.	60	60	1	7137	5014	*TL*	8317	5037	*TL*	**8083**	7572	7389	*TL*	
UDS	L.G.	60	60	2	7144	5474	*TL*	8212	5940	*TL*	**7910**	7137	7128	*TL*	
UDS	L.G.	60	60	3	7779	6699	*TL*	9591	5736	*TL*	**8991**	8242	8164	*TL*	
UDS	L.G.	60	60	4	7240	5723	*TL*	8332	6253	*TL*	**8235**	7807	7724	*TL*	
UDS	L.G.	60	60	5	7017	5615	*TL*	7974	5842	*TL*	**7591**	7112	7062	*TL*	
UDS	L.G.	60	60	6	6919	5236	*TL*	8413	4314	*TL*	**8008**	7284	7270	*TL*	
UDS	L.G.	60	60	7	7293	6011	*TL*	8337	5988	*TL*	**8224**	7691	7647	*TL*	
UDS	L.G.	60	60	8	7524	5430	*TL*	8136	6504	*TL*	**8031**	7584	7548	*TL*	
UDS	L.G.	60	60	9	7048	4352	*TL*	8139	5219	*TL*	**7677**	7121	7083	*TL*	
UDS	L.G.	60	75	0	5923	3996	*TL*	6751	1410	*TL*	**6433**	5675	5622	*TL*	
UDS	L.G.	60	75	1	6992	5091	*TL*	7916	5178	*TL*	**7765**	6938	6827	*TL*	
UDS	L.G.	60	75	2	6459	4733	*TL*	7767	3690	*TL*	**7420**	6421	6418	*TL*	
UDS	L.G.	60	75	3	6513	4765	*TL*	7346	4747	*TL*	**7064**	6322	6266	*TL*	
UDS	L.G.	60	75	4	6076	3867	*TL*	7638	5164	*TL*	**7104**	6311	6181	*TL*	
UDS	L.G.	60	75	5	5487	1941	*TL*	6363	2018	*TL*	**5843**	5276	5207	*TL*	
UDS	L.G.	60	75	6	5978	3101	*TL*	7077	4538	*TL*	**6693**	5784	5769	*TL*	
UDS	L.G.	60	75	7	6997	4948	*TL*	7658	6144	*TL*	**7590**	6974	6897	*TL*	
UDS	L.G.	60	75	8	6382	4289	*TL*	6959	4615	*TL*	**6867**	6168	6041	*TL*	
UDS	L.G.	60	75	9	5633	4193	*TL*	6686	4596	*TL*	**6384**	5632	5544	*TL*	
UDS	L.G.	70	30	0	13755	13747	*TL*	14964	14964	*TL*		16273	16266	*TL*	
UDS	L.G.	70	30	1	15004	15004	2365.6	16517	16517	1961.2		17355	17355	65.8	
UDS	L.G.	70	30	2	15560	15554	*TL*	16916	16826	*TL*		18193	18123	*TL*	
UDS	L.G.	70	30	3	14650	14017	*TL*	16046	16002	*TL*		17199	17199	7.0	
UDS	L.G.	70	30	4	16186	16186	216.0	17600	17600	497.9		18803	18803	1.7	
UDS	L.G.	70	30	5	17132	17132	559.4	18762	18762	842.5		20368	20368	1.8	
UDS	L.G.	70	30	6	16664	16586	*TL*	18073	18073	*TL*		19524	19524	67.1	
UDS	L.G.	70	30	7	16302	16302	136.3	17667	17667	1151.9		19012	19012	1.4	
UDS	L.G.	70	30	8	15871	15871	*TL*	17152	17146	*TL*		18417	18381	*TL*	
UDS	L.G.	70	30	9	15326	15326	302.7	16453	16453	1286.0		18544	18544	1.9	
UDS	L.G.	70	45	0	10639	9759	*TL*	12087	10412	*TL*		11788	11784	*TL*	
UDS	L.G.	70	45	1	11159	8569	*TL*	12624	9535	*TL*		12142	12132	*TL*	
UDS	L.G.	70	45	2	11027	9852	*TL*	12537	9921	*TL*		12201	12137	*TL*	
UDS	L.G.	70	45	3	10218	8188	*TL*	11463	9535	*TL*		10961	10959	*TL*	
UDS	L.G.	70	45	4	10729	9096	*TL*	12089	9704	*TL*		11277	11273	*TL*	
UDS	L.G.	70	45	5	11081	10218	*TL*	12530	10624	*TL*		11987	11987	883.1	
UDS	L.G.	70	45	6	10803	9471	*TL*	12290	10974	*TL*		11655	11655	594.1	
UDS	L.G.	70	45	7	11454	10110	*TL*	13019	10716	*TL*		12658	12658	580.7	
UDS	L.G.	70	45	8	10381	8946	*TL*	11664	10484	*TL*		11450	11427	*TL*	
UDS	L.G.	70	45	9	10153	8651	*TL*	11323	9637	*TL*		11184	11184	392.1	
UDS	L.G.	70	60	0	8721	6715	*TL*	9940	6235	*TL*		9105	9050	*TL*	
UDS	L.G.	70	60	1	9711	6867	*TL*	10719	8133	*TL*		9741	9598	*TL*	
UDS	L.G.	70	60	2	8233	6721	*TL*	9548	6322	*TL*		8757	8670	*TL*	
UDS	L.G.	70	60	3	7769	5228	*TL*	9300	6061	*TL*		8062	7982	*TL*	
UDS	L.G.	70	60	4	9090	5637	*TL*	10558	5807	*TL*		8826	8817	*TL*	
UDS	L.G.	70	60	5	8604	5900	*TL*	9950	6143	*TL*		8734	8708	*TL*	
UDS	L.G.	70	60	6	9280	7285	*TL*	10625	7330	*TL*		9389	9357	*TL*	
UDS	L.G.	70	60	7	8737	6402	*TL*	9664	5750	*TL*		8748	8681	*TL*	
UDS	L.G.	70	60	8	9077	6351	*TL*	10988	8057	*TL*		9151	9122	*TL*	
UDS	L.G.	70	60	9	8176		*TL*	9367	5853	*TL*		8634	8535	*TL*	
UDS	L.G.	70	75	0	7023	3626	*TL*	8230	1298	*TL*		6858	6850	*TL*	
UDS	L.G.	70	75	1	7227	4633	*TL*	8566	5290	*TL*		7345	7240	*TL*	
UDS	L.G.	70	75	2	7356	4508	*TL*	8705	5156	*TL*		7411	7266	*TL*	
UDS	L.G.	70	75	3	7387	4131	*TL*	8614	4437	*TL*		7325	7272	*TL*	

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Instance					Policy										
					optimal			largest gap				traversal			
					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
UDS	L.G.	70	75	4	7021	4615	*TL*	8359	1343	*TL*		7212	7072	*TL*	
UDS	L.G.	70	75	5	6862	4889	*TL*	8643	2045	*TL*		7293	7100	*TL*	
UDS	L.G.	70	75	6	6799	5003	*TL*	8114	3902	*TL*		7093	6912	*TL*	
UDS	L.G.	70	75	7	6704	4029	*TL*	8031	1068	*TL*		6756	6713	*TL*	
UDS	L.G.	70	75	8	6981	3173	*TL*	8354	2682	*TL*		6966	6726	*TL*	
UDS	L.G.	70	75	9	6890	4240	*TL*	8131	2439	*TL*		6833	6802	*TL*	
UDS	L.G.	80	30	0	17161	17161	2521.0	**18583**	18562	*TL*	18608	20387	20387	1037.6	
UDS	L.G.	80	30	1	15900	15897	*TL*	**17274**	17273	*TL*	17284	18427	18427	681.8	
UDS	L.G.	80	30	2	16372	16372	2894.0	**17896**	17896	*TL*	17910	19084	19084	17.9	
UDS	L.G.	80	30	3	18538	18538	320.1	**20325**	20325	2163.4	20500	21943	21943	2.1	
UDS	L.G.	80	30	4	17328	17328	3066.5	18853	18816	*TL*	18853	20096	20043	*TL*	
UDS	L.G.	80	30	5	16514	15829	*TL*	**17842**	17283	*TL*	17929	19252	19252	5.2	
UDS	L.G.	80	30	6	16853	16807	*TL*	**18130**	18100	*TL*	18143	19294	19294	115.4	
UDS	L.G.	80	30	7	16316	16316	1839.8	**17792**	17459	*TL*	17824	18782	18782	44.9	
UDS	L.G.	80	30	8	16687	14927	*TL*	**18303**	18303	2975.5	18454	19302	19302	9.9	
UDS	L.G.	80	30	9	18503	18498	*TL*	**20045**	20045	2544.8	20167	21593	21593	51.3	
UDS	L.G.	80	45	0	11872	9779	*TL*	**13113**	10118	*TL*	13154	12781	12764	*TL*	
UDS	L.G.	80	45	1	12146	10284	*TL*	13704	10821	*TL*	**13646**	13154	13141	*TL*	
UDS	L.G.	80	45	2	11922	9691	*TL*	13418	9462	*TL*	**13404**	13128	13096	*TL*	
UDS	L.G.	80	45	3	11697	9827	*TL*	13262	9822	*TL*	**13231**	12631	12618	*TL*	
UDS	L.G.	80	45	4	12737	10944	*TL*	14386	11229	*TL*	**14304**	13900	13900	1182.8	
UDS	L.G.	80	45	5	12108	10595	*TL*	**13813**	11400	*TL*	13824	13130	13116	*TL*	
UDS	L.G.	80	45	6	11477	9803	*TL*	13177	9337	*TL*	**12923**	12697	12695	*TL*	
UDS	L.G.	80	45	7	13123	10395	*TL*	14658	11537	*TL*	**14641**	14042	14031	*TL*	
UDS	L.G.	80	45	8	11472	10072	*TL*	12923	9451	*TL*	**12756**	12533	12510	*TL*	
UDS	L.G.	80	45	9	11593		*TL*	13352	10062	*TL*	**12952**	12565	12508	*TL*	
UDS	L.G.	80	60	0	10068	7072	*TL*	11578	6365	*TL*	**11062**	10308	10279	*TL*	
UDS	L.G.	80	60	1	9663	7545	*TL*	11132	6209	*TL*	**10766**	9839	9821	*TL*	
UDS	L.G.	80	60	2	10662	8307	*TL*	12423	8504	*TL*	**11949**	10910	10873	*TL*	
UDS	L.G.	80	60	3	9156	6773	*TL*	10717	3679	*TL*	**10445**	9698	9584	*TL*	
UDS	L.G.	80	60	4	9080	6728	*TL*	10600	5007	*TL*	**10427**	9667	9629	*TL*	
UDS	L.G.	80	60	5	10085	7927	*TL*	11755	7680	*TL*	**11698**	10761	10677	*TL*	
UDS	L.G.	80	60	6	10045	7721	*TL*	11158	6026	*TL*	**11020**	10227	10196	*TL*	
UDS	L.G.	80	60	7	10151	7721	*TL*	11836	9123	*TL*	**11778**	10771	10715	*TL*	
UDS	L.G.	80	60	8	10743	8848	*TL*	12613	9887	*TL*	**12506**	11350	11294	*TL*	
UDS	L.G.	80	60	9	9573	7285	*TL*	11496	7872	*TL*	**10942**	10113	10000	*TL*	
UDS	L.G.	80	75	0	8427	4086	*TL*	9875	1388	*TL*	**9420**	8372	8283	*TL*	
UDS	L.G.	80	75	1	7496	4709	*TL*	8709	3482	*TL*	**8410**	7554	7326	*TL*	
UDS	L.G.	80	75	2	7756	4412	*TL*	9496	3810	*TL*	**8928**	7859	7776	*TL*	
UDS	L.G.	80	75	3	8263	5630	*TL*	9904	3883	*TL*	**9297**	8780	8250	*TL*	
UDS	L.G.	80	75	4	7370	4839	*TL*	8835		*TL*	**8611**	7652	7502	*TL*	
UDS	L.G.	80	75	5	8434	4901	*TL*	9842	2467	*TL*	**9302**	8328	8202	*TL*	
UDS	L.G.	80	75	6	8479	6349	*TL*	9950	5534	*TL*	**9776**	9158	8703	*TL*	
UDS	L.G.	80	75	7	8134	5957	*TL*	9699	769	*TL*	**9473**	8339	8233	*TL*	
UDS	L.G.	80	75	8	7921	4705	*TL*	9847	3311	*TL*	**9304**	8219	8016	*TL*	
UDS	L.G.	80	75	9	8345	4626	*TL*	9790	3748	*TL*	**9356**	8718	8312	*TL*	
UDS	L.G.	90	30	0	18761	18090	*TL*	20522	19774	*TL*		21822	21815	*TL*	
UDS	L.G.	90	30	1	20958	20958	1258.1	23041	23041	2714.1		24779	24779	12.6	
UDS	L.G.	90	30	2	20329	20329	654.9	22375	22375	*TL*		24162	24162	25.1	
UDS	L.G.	90	30	3	19016	18979	*TL*	20762	20749	*TL*		22324	22286	*TL*	
UDS	L.G.	90	30	4	19117	18991	*TL*	20883	20843	*TL*		22038	22038	64.4	
UDS	L.G.	90	30	5	18956	18418	*TL*	20610	19925	*TL*		21998	21996	*TL*	
UDS	L.G.	90	30	6	16615	15074	*TL*	18059	15171	*TL*		19208	19208	216.3	
UDS	L.G.	90	30	7	18882	18327	*TL*	20625	20507	*TL*		21841	21841	80.8	
UDS	L.G.	90	30	8	19299	19299	3157.0	20899	20882	*TL*		22621	22621	6.2	
UDS	L.G.	90	30	9	19805	19805	*TL*	21438	21430	*TL*		23175	23141	*TL*	
UDS	L.G.	90	45	0	12104	9691	*TL*	13573	12163	*TL*		13262	13229	*TL*	
UDS	L.G.	90	45	1	12473	10106	*TL*	13977	10132	*TL*		13492	13430	*TL*	
UDS	L.G.	90	45	2	13235	8834	*TL*	14707	11495	*TL*		14161	14155	*TL*	
UDS	L.G.	90	45	3	13209		*TL*	14835	11125	*TL*		14491	14468	*TL*	
UDS	L.G.	90	45	4	14008	12141	*TL*	15884	12776	*TL*		15228	15228	1460.7	
UDS	L.G.	90	45	5	13048	10845	*TL*	14756	11542	*TL*		14163	14122	*TL*	
UDS	L.G.	90	45	6	13292		*TL*	15109	11045	*TL*		14606	14568	*TL*	
UDS	L.G.	90	45	7	13359	10893	*TL*	15049	12434	*TL*		14474	14459	*TL*	
UDS	L.G.	90	45	8	12609	10002	*TL*	14144	12508	*TL*		13745	13720	*TL*	
UDS	L.G.	90	45	9	13863	12609	*TL*	15806	10909	*TL*		15082	15062	*TL*	
UDS	L.G.	90	60	0	10422	7118	*TL*	12060	8248	*TL*		10798	10623	*TL*	
UDS	L.G.	90	60	1	10216	7841	*TL*	11984	7205	*TL*		10704	10641	*TL*	
UDS	L.G.	90	60	2	11197	7465	*TL*	12934	6398	*TL*		11472	11440	*TL*	
UDS	L.G.	90	60	3	12219	9291	*TL*	14092	9484	*TL*		12507	12466	*TL*	

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					Policy											
					optimal			largest gap				traversal				
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS	
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB	
UDS	L.G.	90	60	4	11021	7937	TL	12757	6644	TL		11823	11411	TL		
UDS	L.G.	90	60	5	10991	7614	TL	12531	6226	TL		11353	11244	TL		
UDS	L.G.	90	60	6	11634	8811	TL	12978	8773	TL		11900	11786	TL		
UDS	L.G.	90	60	7	11757	9200	TL	13650	7938	TL		12374	11931	TL		
UDS	L.G.	90	60	8	10970	8028	TL	12213	7661	TL		11324	11188	TL		
UDS	L.G.	90	60	9	12018		TL	13515	7886	TL		11756	11738	TL		
UDS	L.G.	90	75	0	8821	5163	TL	10562	975	TL		9003	8628	TL		
UDS	L.G.	90	75	1	9142	6101	TL	10862	2007	TL		9453	9285	TL		
UDS	L.G.	90	75	2	8796	3782	TL	10298	3297	TL		9205	8748	TL		
UDS	L.G.	90	75	3	9311	5882	TL	11192	4760	TL		9259	9039	TL		
UDS	L.G.	90	75	4	9645	7120	TL	11756	3326	TL		9955	9869	TL		
UDS	L.G.	90	75	5	8920	5018	TL	10578		TL		9219	8740	TL		
UDS	L.G.	90	75	6	8828	5487	TL	10938	2643	TL		9245	9014	TL		
UDS	L.G.	90	75	7	8698	4810	TL	10458	510	TL		8941	8457	TL		
UDS	L.G.	90	75	8	8275	3969	TL	9908		TL		8016	7888	TL		
UDS	L.G.	90	75	9	8991	5618	TL	10808	5364	TL		9186	9080	TL		
UDS	L.G.	100	30	0	22688	22688	1427.7	**24773**	24773	2808.0	24778	26373	26373	46.9		
UDS	L.G.	100	30	1	18739	17961	TL	**20473**	17875	TL	20528	22087	22087	398.7		
UDS	L.G.	100	30	2	22202	22202	607.5	**24320**	24318	TL	24323	25982	25982	2.6		
UDS	L.G.	100	30	3	20721	20178	TL	22708	22157	TL	**22707**	24322	24314	TL		
UDS	L.G.	100	30	4	19804	17888	TL	**21474**	21473	TL	21610	22762	22762	152.7		
UDS	L.G.	100	30	5	24129	24129	974.0	**26263**	26263	1041.0	26472	28541	28541	3.1		
UDS	L.G.	100	30	6	22018	21969	TL	24073	24022	TL	**24066**	26146	26146	64.8		
UDS	L.G.	100	30	7	20674	20629	TL	**22574**	22535	TL	22596	24005	24005	155.5		
UDS	L.G.	100	30	8	20105	19141	TL	**21837**	21109	TL	21841	23351	23351	139.4		
UDS	L.G.	100	30	9	22007	22005	TL	**23884**	23637	TL	23910	25775	25764	TL		
UDS	L.G.	100	45	0	13980		TL	15860	13049	TL	**15846**	15458	15428	TL		
UDS	L.G.	100	45	1	14360	11752	TL	16330	11777	TL	**16243**	15443	15425	TL		
UDS	L.G.	100	45	2	15607	13244	TL	17541	13315	TL	**17500**	16840	16832	TL		
UDS	L.G.	100	45	3	14866	12043	TL	16727	13339	TL	**16650**	16243	16238	TL		
UDS	L.G.	100	45	4	15521	12488	TL	17499	13171	TL	**17462**	16887	16876	TL		
UDS	L.G.	100	45	5	15192	12088	TL	17009	13022	TL	**16985**	16231	16219	TL		
UDS	L.G.	100	45	6	15784		TL	**17421**	13758	TL	17500	17394	17368	TL		
UDS	L.G.	100	45	7	15102	13125	TL	**16965**	13844	TL	17006	16834	16824	TL		
UDS	L.G.	100	45	8	15324	12174	TL	17500	12933	TL	**17305**	16654	16637	TL		
UDS	L.G.	100	45	9	16066	13832	TL	18204	13940	TL	**18174**	17539	17525	TL		
UDS	L.G.	100	60	0	12973	10090	TL	15017	10526	TL	**14956**	13940	13863	TL		
UDS	L.G.	100	60	1	12261		TL	14085	7893	TL	**13730**	12586	12512	TL		
UDS	L.G.	100	60	2	12535	9062	TL	14175	6274	TL	**13897**	12801	12423	TL		
UDS	L.G.	100	60	3	12696	8423	TL	14805	6215	TL	**14230**	13316	13191	TL		
UDS	L.G.	100	60	4	12483	9473	TL	13982	8394	TL	**13803**	12742	12537	TL		
UDS	L.G.	100	60	5	12059	7437	TL	13687	3949	TL	**13356**	12597	12081	TL		
UDS	L.G.	100	60	6	11349		TL	12956	749	TL	**12683**	11737	11582	TL		
UDS	L.G.	100	60	7	12302	8290	TL	13516	7865	TL	**13184**	12292	12215	TL		
UDS	L.G.	100	60	8	11967		TL	13174	7724	TL	**12906**	12058	11949	TL		
UDS	L.G.	100	60	9	11897	8976	TL	13314	6762	TL	**13134**	12225	12022	TL		
UDS	L.G.	100	75	0	9846	5380	TL	11560	2737	TL	**11056**	10300	9841	TL		
UDS	L.G.	100	75	1	9195	3530	TL	10962	171	TL	**10277**	9550	9043	TL		
UDS	L.G.	100	75	2	9764	7082	TL	11450	5663	TL	**11146**	10105	9883	TL		
UDS	L.G.	100	75	3	9671	6261	TL	11711	1943	TL	**11016**	10130	9524	TL		
UDS	L.G.	100	75	4	9394	3118	TL	10851		TL	**10279**	9631	9155	TL		
UDS	L.G.	100	75	5	10369		TL	12237	3591	TL	**12049**	10973	10668	TL		
UDS	L.G.	100	75	6	10000	5449	TL	11821	4008	TL	**11277**	10291	9792	TL		
UDS	L.G.	100	75	7	9565	5616	TL	11231	2127	TL	**10680**	9883	9524	TL		
UDS	L.G.	100	75	8	9992	5492	TL	11829	5217	TL	**11209**	10345	9781	TL		
UDS	L.G.	100	75	9	9672	5908	TL	11548	112	TL	**10976**	9958	9442	TL		
UDS	S-S.	20	30	0	5067	5067	24.3	5468	5468	2.5		6144	6144	0.8		
UDS	S-S.	20	30	1	4536	4536	9.6	4973	4973	3.4		5225	5225	0.6		
UDS	S-S.	20	30	2	4409	4409	16.8	4868	4868	1.0		5277	5277	0.4		
UDS	S-S.	20	30	3	5091	5091	3.3	5591	5591	4.9		6051	6051	0.8		
UDS	S-S.	20	30	4	4035	4035	22.3	4387	4387	14.4		4641	4641	1.3		
UDS	S-S.	20	30	5	4630	4630	4.2	5024	5024	2.6		5491	5491	0.5		
UDS	S-S.	20	30	6	5527	5527	26.4	6074	6074	8.3		6513	6513	1.7		
UDS	S-S.	20	30	7	4530	4530	10.0	4876	4876	1.5		5311	5311	0.7		
UDS	S-S.	20	30	8	4435	4435	113.3	4780	4780	15.9		5068	5068	120.3		
UDS	S-S.	20	30	9	3677	3677	251.8	3918	3918	6.0		4326	4326	2.0		
UDS	S-S.	20	45	0	3674	3674	411.4	4142	4142	8.6		4155	4155	25.8		
UDS	S-S.	20	45	1	2909	2888	TL	3290	3290	2327.5		3184	3172	TL		
UDS	S-S.	20	45	2	3513	3513	337.0	3961	3961	70.6		3774	3774	1.7		
UDS	S-S.	20	45	3	3294	3258	TL	3666	3666	568.7		3599	3599	55.5		

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Instance					Policy										
					optimal			largest gap				traversal			
					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
UDS	S-S.	20	45	4	3425	3425	25.2	3933	3933	28.6		3624	3624	11.9	
UDS	S-S.	20	45	5	4183	4183	336.9	4635	4635	56.3		4763	4763	5.1	
UDS	S-S.	20	45	6	3291	3284	*TL*	3761	3761	342.7		3617	3617	364.8	
UDS	S-S.	20	45	7	2825	2825	819.4	3151	3151	66.3		3176	3176	1316.6	
UDS	S-S.	20	45	8	3842	3842	2327.6	4213	4213	85.2		4251	4251	312.1	
UDS	S-S.	20	45	9	3077	3077	62.7	3538	3538	146.2		3415	3415	1.4	
UDS	S-S.	20	60	0	2936	2865	*TL*	3334	3309	*TL*		3132	3105	*TL*	
UDS	S-S.	20	60	1	2466	2430	*TL*	2832	2832	1103.4		2678	2657	*TL*	
UDS	S-S.	20	60	2	3016	3016	437.1	3529	3529	47.7		3285	3285	61.4	
UDS	S-S.	20	60	3	2455	2438	*TL*	2852	2852	718.1		2671	2646	*TL*	
UDS	S-S.	20	60	4	2475	2462	*TL*	2869	2869	2125.9		2662	2634	*TL*	
UDS	S-S.	20	60	5	2342	2315	*TL*	2677	2677	1783.6		2528	2528	935.4	
UDS	S-S.	20	60	6	2481	2430	*TL*	2830	2824	*TL*		2616	2562	*TL*	
UDS	S-S.	20	60	7	2466	2378	*TL*	2848	2775	*TL*		2720	2527	*TL*	
UDS	S-S.	20	60	8	2348	2285	*TL*	2680	2671	*TL*		2526	2464	*TL*	
UDS	S-S.	20	60	9	2475	2475	562.1	2874	2874	213.1		2645	2645	6.5	
UDS	S-S.	20	75	0	2058	2058	145.8	2516	2516	134.7		2203	2192	*TL*	
UDS	S-S.	20	75	1	2086	2086	353.4	2532	2532	160.9		2191	2191	153.9	
UDS	S-S.	20	75	2	2100	2100	1770.4	2540	2540	115.1		2209	2196	*TL*	
UDS	S-S.	20	75	3	2054	1977	*TL*	2407	2404	*TL*		2157	2078	*TL*	
UDS	S-S.	20	75	4	2530	2444	*TL*	2955	2918	*TL*		2791	2574	*TL*	
UDS	S-S.	20	75	5	2082	2033	*TL*	2461	2440	*TL*		2186	2138	*TL*	
UDS	S-S.	20	75	6	2066	2014	*TL*	2437	2437	3160.6		2211	2126	*TL*	
UDS	S-S.	20	75	7	2317	2317	1371.4	2775	2775	282.2		2445	2445	320.4	
UDS	S-S.	20	75	8	2045	2045	*TL*	2453	2453	1859.6		2202	2163	*TL*	
UDS	S-S.	20	75	9	2071	2033	*TL*	2503	2491	*TL*		2213	2132	*TL*	
UDS	S-S.	30	30	0	7714	7714	46.6	8421	8421	6.4		9299	9299	3.8	
UDS	S-S.	30	30	1	7353	7353	105.3	8053	8053	8.0		8783	8783	8.8	
UDS	S-S.	30	30	2	9492	9492	82.1	10240	10240	11.8		11367	11367	3.6	
UDS	S-S.	30	30	3	6972	6972	77.1	7623	7623	39.8		8125	8125	3.8	
UDS	S-S.	30	30	4	6613	6613	33.8	7204	7204	2.6		7816	7816	1.0	
UDS	S-S.	30	30	5	6682	6682	56.3	7110	7110	11.1		7752	7752	0.7	
UDS	S-S.	30	30	6	6888	6888	259.5	7577	7577	143.7		8205	8205	4.2	
UDS	S-S.	30	30	7	7069	7069	20.1	7646	7646	6.7		8260	8260	0.7	
UDS	S-S.	30	30	8	5677	5677	72.9	6198	6198	19.7		6538	6538	2.9	
UDS	S-S.	30	30	9	7363	7363	14.1	7980	7980	7.2		8889	8889	0.9	
UDS	S-S.	30	45	0	4667	4578	*TL*	5209	5209	2842.5		5051	5051	1107.8	
UDS	S-S.	30	45	1	5664	5664	674.8	6358	6358	121.3		6281	6281	1020.5	
UDS	S-S.	30	45	2	4797	4797	3007.0	5388	5388	264.4		5291	5291	461.0	
UDS	S-S.	30	45	3	4306	4254	*TL*	4846	4844	*TL*		4657	4644	*TL*	
UDS	S-S.	30	45	4	5198	5184	*TL*	5798	5798	138.0		5735	5735	157.9	
UDS	S-S.	30	45	5	4671	4651	*TL*	5338	5338	566.8		5107	5102	*TL*	
UDS	S-S.	30	45	6	5230	5224	*TL*	5886	5886	637.2		5661	5661	97.2	
UDS	S-S.	30	45	7	5259	5259	2639.1	5922	5922	863.4		5712	5712	2396.3	
UDS	S-S.	30	45	8	5202	5202	397.6	5854	5854	1300.2		5892	5892	418.0	
UDS	S-S.	30	45	9	4491	4468	*TL*	5033	5033	2341.5		4982	4982	788.2	
UDS	S-S.	30	60	0	3905	3853	*TL*	4526	4495	*TL*		4277	4247	*TL*	
UDS	S-S.	30	60	1	3995	3838	*TL*	4611	4531	*TL*		4238	4138	*TL*	
UDS	S-S.	30	60	2	3334	3334	675.6	3886	3868	*TL*		3643	3643	2206.4	
UDS	S-S.	30	60	3	3953	3825	*TL*	4365	4365	1579.5		4195	4127	*TL*	
UDS	S-S.	30	60	4	3939	3880	*TL*	4499	4478	*TL*		4159	4122	*TL*	
UDS	S-S.	30	60	5	3411	3340	*TL*	3930	3908	*TL*		3643	3640	*TL*	
UDS	S-S.	30	60	6	3881	3802	*TL*	4431	4382	*TL*		4293	4253	*TL*	
UDS	S-S.	30	60	7	3455	3447	*TL*	4034	4034	743.4		3727	3723	*TL*	
UDS	S-S.	30	60	8	3916	3755	*TL*	4475	4367	*TL*		4180	4051	*TL*	
UDS	S-S.	30	60	9	3488	3421	*TL*	4011	4011	1017.6		3710	3710	103.1	
UDS	S-S.	30	75	0	3099	3016	*TL*	3583	3556	*TL*		3304	3215	*TL*	
UDS	S-S.	30	75	1	3060	2780	*TL*	3663	3388	*TL*		3229	2994	*TL*	
UDS	S-S.	30	75	2	3529	3370	*TL*	4250	4087	*TL*		3717	3616	*TL*	
UDS	S-S.	30	75	3	2595	2595	3479.6	3152	3152	*TL*		2751	2751	440.5	
UDS	S-S.	30	75	4	3035	2946	*TL*	3583	3544	*TL*		3166	3147	*TL*	
UDS	S-S.	30	75	5	3608	3392	*TL*	4283	4094	*TL*		3746	3593	*TL*	
UDS	S-S.	30	75	6	3058	2991	*TL*	3555	3528	*TL*		3222	3204	*TL*	
UDS	S-S.	30	75	7	3289	3288	*TL*	3956	3951	*TL*		3541	3541	168.9	
UDS	S-S.	30	75	8	3091	3091	2566.7	3744	3739	*TL*		3243	3243	154.3	
UDS	S-S.	30	75	9	3054	2969	*TL*	3626	3547	*TL*		3301	3204	*TL*	
UDS	S-S.	40	30	0	9246	9246	51.3	10012	10012	69.6		10751	10751	1.0	10751
UDS	S-S.	40	30	1	7956	7956	1038.3	8686	8686	1585.1		**9459**	9459	20.1	9561
UDS	S-S.	40	30	2	9448	9448	18.6	10432	10432	13.1		11012	11012	0.8	11012
UDS	S-S.	40	30	3	11307	11307	171.0	12227	12227	102.6		13471	13471	1.1	13471

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					Policy										
					optimal			largest gap				traversal			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
UDS	S-S.	40	30	4	9400	9400	100.9	10332	10332	41.6		**11145**	11145	2.7	11148
UDS	S-S.	40	30	5	8617	8617	91.6	9299	9299	130.3		10223	10223	44.6	10223
UDS	S-S.	40	30	6	10157	10157	140.0	11073	11073	216.5		11546	11546	5.0	11546
UDS	S-S.	40	30	7	9421	9421	50.9	10322	10322	325.3		11300	11300	1.1	11300
UDS	S-S.	40	30	8	9523	9523	18.1	10275	10275	141.3		11167	11167	2.7	11167
UDS	S-S.	40	30	9	8704	8671	*TL*	9723	9723	298.0		10040	10040	14.1	10040
UDS	S-S.	40	45	0	6093	6061	*TL*	6908	6879	*TL*		**6749**	6743	*TL*	6755
UDS	S-S.	40	45	1	7022	6972	*TL*	7940	7930	*TL*		**7781**	7734	*TL*	7788
UDS	S-S.	40	45	2	6834	6698	*TL*	7673	7576	*TL*		**7514**	7514	38.8	7638
UDS	S-S.	40	45	3	6659	6602	*TL*	7427	7421	*TL*		**7361**	7361	1795.6	7369
UDS	S-S.	40	45	4	6393	5998	*TL*	6968	6958	*TL*		**6854**	6854	90.2	6873
UDS	S-S.	40	45	5	5936	5902	*TL*	6624	6623	*TL*		**6644**	6620	*TL*	6653
UDS	S-S.	40	45	6	6165	6165	1861.4	6977	6970	*TL*		6698	6698	15.6	6698
UDS	S-S.	40	45	7	5590	5551	*TL*	6220	6216	*TL*		**6128**	6128	540.4	6150
UDS	S-S.	40	45	8	6117	6069	*TL*	6913	6899	*TL*		6748	6700	*TL*	6748
UDS	S-S.	40	45	9	6057	5936	*TL*	6862	6760	*TL*		**6609**	6536	*TL*	6616
UDS	S-S.	40	60	0	5555	5422	*TL*	6911	6454	*TL*		**5839**	5826	*TL*	5846
UDS	S-S.	40	60	1	4848	4676	*TL*	5654	5561	*TL*		**5234**	5153	*TL*	5240
UDS	S-S.	40	60	2	5221	4718	*TL*	5785	5493	*TL*		**5250**	5214	*TL*	5252
UDS	S-S.	40	60	3	5416	4899	*TL*	6296	5315	*TL*		5666	5587	*TL*	**5658**
UDS	S-S.	40	60	4	5933	5509	*TL*	6433	6185	*TL*		**5934**	5923	*TL*	5947
UDS	S-S.	40	60	5	4983	4846	*TL*	5899	5635	*TL*		5259	5192	*TL*	5259
UDS	S-S.	40	60	6	4794	4257	*TL*	5639	5178	*TL*		**5129**	5027	*TL*	5135
UDS	S-S.	40	60	7	4840	4188	*TL*	5618	5061	*TL*		**4770**	4770	834.6	4794
UDS	S-S.	40	60	8	4965	4557	*TL*	5715	5530	*TL*		**5234**	5155	*TL*	5236
UDS	S-S.	40	60	9	4879	4469	*TL*	5704	5635	*TL*		5115	5056	*TL*	**5104**
UDS	S-S.	40	75	0	4568	4288	*TL*	5526	4341	*TL*		4900	4628	*TL*	**4639**
UDS	S-S.	40	75	1	4195	3615	*TL*	5448	4337	*TL*		4312	4284	*TL*	**4309**
UDS	S-S.	40	75	2	4472	4144	*TL*	5369	5081	*TL*		4819	4724	*TL*	**4800**
UDS	S-S.	40	75	3	3947	3772	*TL*	4747	4445	*TL*		4219	4031	*TL*	**4154**
UDS	S-S.	40	75	4	4705	4199	*TL*	5361	4611	*TL*		4805	4603	*TL*	**4700**
UDS	S-S.	40	75	5	4039	3534	*TL*	4709	4135	*TL*		**4172**	4138	*TL*	4182
UDS	S-S.	40	75	6	3956	3495	*TL*	4710	4228	*TL*		4262	4148	*TL*	**4218**
UDS	S-S.	40	75	7	4075	3699	*TL*	5330	4703	*TL*		**4303**	4303	2361.4	4361
UDS	S-S.	40	75	8	3960	3346	*TL*	4738	4261	*TL*		4263	3991	*TL*	**4077**
UDS	S-S.	40	75	9	3895	3725	*TL*	4714	4304	*TL*		**3980**	3980	22.8	4047
UDS	S-S.	50	30	0	11045	11045	2142.8	12103	12103	1771.7		12954	12954	1687.6	
UDS	S-S.	50	30	1	12551	12551	502.8	13540	13540	1323.7		14501	14501	2.5	
UDS	S-S.	50	30	2	9355	9347	*TL*	10178	10178	2539.7		10874	10874	91.1	
UDS	S-S.	50	30	3	11090	11090	939.6	12071	12065	*TL*		13085	13085	154.0	
UDS	S-S.	50	30	4	10457	10457	1801.0	11383	11383	1104.7		12301	12301	38.3	
UDS	S-S.	50	30	5	11740	11740	68.6	12680	12680	202.9		14175	14175	1.2	
UDS	S-S.	50	30	6	11271	11271	65.7	12250	12250	34.3		13387	13387	1.1	
UDS	S-S.	50	30	7	12822	12822	69.9	13935	13935	81.7		15801	15801	1.2	
UDS	S-S.	50	30	8	11994	11994	650.1	13050	13050	2259.2		14373	14373	*TL*	
UDS	S-S.	50	30	9	11467	11467	64.8	12521	12521	125.2		13243	13243	1.3	
UDS	S-S.	50	45	0	7590	7050	*TL*	8515	8482	*TL*		8070	8070	259.3	
UDS	S-S.	50	45	1	7559	7058	*TL*	8877	8502	*TL*		8376	8376	154.0	
UDS	S-S.	50	45	2	8481	8006	*TL*	9552	9496	*TL*		9440	9440	216.5	
UDS	S-S.	50	45	3	8676	8114	*TL*	9794	9697	*TL*		9498	9456	*TL*	
UDS	S-S.	50	45	4	7466	6976	*TL*	8369	7928	*TL*		8300	8258	*TL*	
UDS	S-S.	50	45	5	7888	7762	*TL*	8766	8101	*TL*		8742	8742	413.1	
UDS	S-S.	50	45	6	8031	7768	*TL*	9118	7990	*TL*		8691	8621	*TL*	
UDS	S-S.	50	45	7	8425	8384	*TL*	9501	9499	*TL*		9322	9307	*TL*	
UDS	S-S.	50	45	8	7843	7539	*TL*	8792	8775	*TL*		8577	8577	2696.6	
UDS	S-S.	50	45	9	8454	7267	*TL*	9501	9373	*TL*		9470	9459	*TL*	
UDS	S-S.	50	60	0	6423	5894	*TL*	7732	6861	*TL*		6857	6857	116.1	
UDS	S-S.	50	60	1	6224	5632	*TL*	7253	5582	*TL*		6504	6492	*TL*	
UDS	S-S.	50	60	2	5819	4931	*TL*	6705	6035	*TL*		6043	6029	*TL*	
UDS	S-S.	50	60	3	6115	4622	*TL*	7040	5731	*TL*		6196	6188	*TL*	
UDS	S-S.	50	60	4	6841	5652	*TL*	7991	6744	*TL*		7227	7227	1983.4	
UDS	S-S.	50	60	5	6743	5282	*TL*	7772	5790	*TL*		6870	6852	*TL*	
UDS	S-S.	50	60	6	7260	5841	*TL*	8622	7056	*TL*		7539	7524	*TL*	
UDS	S-S.	50	60	7	6243	5450	*TL*	7655	5949	*TL*		6666	6560	*TL*	
UDS	S-S.	50	60	8	5899	4872	*TL*	6772	5741	*TL*		6203	6181	*TL*	
UDS	S-S.	50	60	9	6335	5227	*TL*	7415	5477	*TL*		6882	6737	*TL*	
UDS	S-S.	50	75	0	4989	4134	*TL*	6573	4481	*TL*		5261	5243	*TL*	
UDS	S-S.	50	75	1	5441	4229	*TL*	6473	4070	*TL*		5312	5258	*TL*	
UDS	S-S.	50	75	2	5300	4307	*TL*	6464	4259	*TL*		5364	5302	*TL*	
UDS	S-S.	50	75	3	5530	4039	*TL*	6612	4005	*TL*		5365	5343	*TL*	

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					Policy										
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Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\lvert O\rvert$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
UDS	S-S.	50	75	4	5183	3969	TL	6464	4434	TL		5321	5260	TL	
UDS	S-S.	50	75	5	5438	4389	TL	6575	4266	TL		5219	5175	TL	
UDS	S-S.	50	75	6	4982	3869	TL	6437	4167	TL		5262	5196	TL	
UDS	S-S.	50	75	7	5273	3553	TL	6451	4466	TL		5288	5239	TL	
UDS	S-S.	50	75	8	5463	4489	TL	6600	4634	TL		5768	5639	TL	
UDS	S-S.	50	75	9	5498	4806	TL	6504	5180	TL		5833	5721	TL	
UDS	S-S.	60	30	0	14621	14621	75.2	15710	15710	350.6		16755	16755	1.4	16755
UDS	S-S.	60	30	1	14731	14731	1569.5	16047	16047	1752.2		17479	17479	15.1	17479
UDS	S-S.	60	30	2	13033	13033	2672.2	14269	14269	2094.7		15158	15158	2.5	15158
UDS	S-S.	60	30	3	13423	13423	420.8	14658	14658	1480.5		15826	15826	1.6	15826
UDS	S-S.	60	30	4	13881	13881	978.8	15025	15025	1200.0		16499	16499	2.9	16499
UDS	S-S.	60	30	5	12394	12394	3498.9	13500	13500	1884.5		14052	14052	22.8	14052
UDS	S-S.	60	30	6	12220	12220	3021.1	13281	13281	2114.4		**14343**	14343	57.3	14389
UDS	S-S.	60	30	7	13453	13453	508.2	14587	14587	966.6		16007	16007	7.7	16007
UDS	S-S.	60	30	8	15542	15542	1222.8	16947	16947	1632.8		18801	18801	145.8	18801
UDS	S-S.	60	30	9	12944	12944	392.7	14113	14113	736.2		15000	15000	6.9	15000
UDS	S-S.	60	45	0	9370	8593	TL	10530	9125	TL		**10268**	10268	1903.3	10295
UDS	S-S.	60	45	1	9034	7649	TL	10079	8909	TL		**9767**	9759	TL	9781
UDS	S-S.	60	45	2	7787	4559	TL	8985	7199	TL		**7987**	7987	2378.6	8085
UDS	S-S.	60	45	3	8428	7248	TL	9704	8594	TL		**9207**	9207	1331.5	9275
UDS	S-S.	60	45	4	9225	8192	TL	10209	9905	TL		**10061**	10060	TL	10145
UDS	S-S.	60	45	5	9295	7793	TL	10575	9622	TL		**9947**	9904	TL	9969
UDS	S-S.	60	45	6	9785	9133	TL	10999	9701	TL		**10756**	10711	TL	10770
UDS	S-S.	60	45	7	9495	8280	TL	10810	9605	TL		**10365**	10365	2366.6	10405
UDS	S-S.	60	45	8	10452	8865	TL	11711	10207	TL		**11123**	11123	519.8	11205
UDS	S-S.	60	45	9	9571	8502	TL	11050	9640	TL		**10439**	10439	897.3	10607
UDS	S-S.	60	60	0	7651	5651	TL	8827	7246	TL		**8272**	8192	TL	8283
UDS	S-S.	60	60	1	7752	6355	TL	8804	6587	TL		8143	8012	TL	8143
UDS	S-S.	60	60	2	7160	6090	TL	8531	6017	TL		**7586**	7569	TL	7603
UDS	S-S.	60	60	3	7265	5535	TL	8601	6072	TL		7766	7650	TL	**7742**
UDS	S-S.	60	60	4	8261	6051	TL	9006	4732	TL		**8014**	8014	1524.5	8106
UDS	S-S.	60	60	5	7769	5106	TL	8281	4930	TL		**7243**	7229	TL	7372
UDS	S-S.	60	60	6	6664	4625	TL	8215	5707	TL		**7104**	7021	TL	7141
UDS	S-S.	60	60	7	6715	5288	TL	7732	4073	TL		**7058**	7024	TL	7141
UDS	S-S.	60	60	8	7733	5724	TL	9018	7025	TL		**7932**	7928	TL	8018
UDS	S-S.	60	60	9	7178	5150	TL	8446	4853	TL		**7118**	7029	TL	7144
UDS	S-S.	60	75	0	5747	4086	TL	6895	3931	TL		**5755**	5707	TL	5792
UDS	S-S.	60	75	1	6603	5066	TL	8036	4032	TL		7049	6836	TL	**7009**
UDS	S-S.	60	75	2	6153	3939	TL	6981	5257	TL		**6240**	6131	TL	6245
UDS	S-S.	60	75	3	6352	4078	TL	7559	3714	TL		**6260**	6212	TL	6294
UDS	S-S.	60	75	4	6019	4017	TL	7100	3446	TL		**5785**	5756	TL	5922
UDS	S-S.	60	75	5	5894	3949	TL	7093	2742	TL		**5736**	5718	TL	5829
UDS	S-S.	60	75	6	5994	2917	TL	7135	4176	TL		**5763**	5760	TL	5862
UDS	S-S.	60	75	7	6006	4551	TL	7753	4541	TL		**6278**	6152	TL	6287
UDS	S-S.	60	75	8	6349	4787	TL	7447	3241	TL		**6263**	6247	TL	6325
UDS	S-S.	60	75	9	6359	4077	TL	7046	4021	TL		**6252**	6198	TL	6257
UDS	S-S.	70	30	0	18190	18190	86.8	19945	19945	735.7		21731	21731	1.9	
UDS	S-S.	70	30	1	15542	15542	83.9	16936	16936	613.9		18663	18663	1.7	
UDS	S-S.	70	30	2	15549	15549	887.5	16997	16997	281.7		18180	18180	1.8	
UDS	S-S.	70	30	3	16809	16809	183.2	18273	18273	TL		19901	19901	2.0	
UDS	S-S.	70	30	4	16149	16149	1539.9	17529	17529	684.3		19110	19110	1.9	
UDS	S-S.	70	30	5	16036	16036	194.7	17598	17598	1806.1		19037	19037	1.7	
UDS	S-S.	70	30	6	14661	14661	2945.5	15941	15941	1611.4		17101	17101	12.5	
UDS	S-S.	70	30	7	14839	14839	TL	16063	15659	TL		17440	17440	1263.1	
UDS	S-S.	70	30	8	17525	17525	392.3	18785	18785	662.9		20942	20942	18.3	
UDS	S-S.	70	30	9	16705	16705	103.4	18389	18389	195.9		20179	20179	1.8	
UDS	S-S.	70	45	0	9598	8366	TL	10749	9890	TL		10622	10599	TL	
UDS	S-S.	70	45	1	11192	9992	TL	12632	10881	TL		12278	12269	TL	
UDS	S-S.	70	45	2	11209	9872	TL	12580	11195	TL		12493	12447	TL	
UDS	S-S.	70	45	3	10350	7940	TL	11681	9339	TL		11250	11247	TL	
UDS	S-S.	70	45	4	11178	10522	TL	12709	11746	TL		12264	12264	1282.8	
UDS	S-S.	70	45	5	10550	9564	TL	11875	10970	TL		11668	11633	TL	
UDS	S-S.	70	45	6	10390	8724	TL	11722	10143	TL		11363	11325	TL	
UDS	S-S.	70	45	7	10741	8964	TL	11680	8783	TL		11203	11203	TL	
UDS	S-S.	70	45	8	10745	9175	TL	12106	9108	TL		11835	11716	TL	
UDS	S-S.	70	45	9	10963	8917	TL	12411	10604	TL		11694	11694	1402.5	
UDS	S-S.	70	60	0	7610	5404	TL	8762	5713	TL		7969	7931	TL	
UDS	S-S.	70	60	1	8607	5873	TL	9984	5518	TL		8729	8729	2288.3	
UDS	S-S.	70	60	2	8771	6401	TL	10204	8270	TL		9208	9128	TL	
UDS	S-S.	70	60	3	8900	6370	TL	9772	7140	TL		8724	8700	TL	

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Instance					Policy										
					optimal			largest gap				traversal			
					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\lvert O\rvert$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
UDS	S-S.	70	60	4	9090	6126	TL	10515	7459	TL		9261	9248	TL	
UDS	S-S.	70	60	5	8170	5400	TL	9305	7157	TL		8613	8438	TL	
UDS	S-S.	70	60	6	9418	7441	TL	10710	5071	TL		9779	9756	TL	
UDS	S-S.	70	60	7	9803	7729	TL	10770	7377	TL		9702	9669	TL	
UDS	S-S.	70	60	8	8064	5716	TL	9411	5989	TL		8625	8535	TL	
UDS	S-S.	70	60	9	8799	7679	TL	10321	7033	TL		9331	9265	TL	
UDS	S-S.	70	75	0	6930	4563	TL	8261	4699	TL		7243	7058	TL	
UDS	S-S.	70	75	1	6875	5152	TL	8320	4799	TL		7167	6982	TL	
UDS	S-S.	70	75	2	6993	2918	TL	8396	1938	TL		6727	6697	TL	
UDS	S-S.	70	75	3	6980	4886	TL	8512	3914	TL		7364	7236	TL	
UDS	S-S.	70	75	4	6904	4143	TL	8350	2344	TL		7135	7019	TL	
UDS	S-S.	70	75	5	6952	4002	TL	8530	2316	TL		7229	7064	TL	
UDS	S-S.	70	75	6	7502	5592	TL	9293	1959	TL		7774	7654	TL	
UDS	S-S.	70	75	7	6962	3922	TL	8155	2210	TL		7137	6721	TL	
UDS	S-S.	70	75	8	6672	4944	TL	8067	2633	TL		7090	6609	TL	
UDS	S-S.	70	75	9	7350		TL	8776	5702	TL		7691	7499	TL	
UDS	S-S.	80	30	0	18930	18930	657.6	20746	20746	1494.8		22711	22711	2.1	22711
UDS	S-S.	80	30	1	18357	18352	TL	20114	20114	TL		21987	21947	TL	21987
UDS	S-S.	80	30	2	16778	16573	TL	18256	18256	3314.4		19650	19632	TL	19650
UDS	S-S.	80	30	3	17961	17961	168.0	19565	19565	1523.0		20792	20792	2.4	20792
UDS	S-S.	80	30	4	19708	19708	346.0	21444	21444	1207.7		23398	23398	20.7	23398
UDS	S-S.	80	30	5	18142	18142	TL	19897	19897	3270.5		21718	21718	28.0	21718
UDS	S-S.	80	30	6	17297	17297	TL	18962	18962	1806.1		19941	19941	4.7	19941
UDS	S-S.	80	30	7	16932	16932	1863.5	18310	18310	108.1		19715	19710	TL	19715
UDS	S-S.	80	30	8	16360	14494	TL	17798	17736	TL		**18762**	18762	16.7	18801
UDS	S-S.	80	30	9	15958	14584	TL	17555	17537	TL		**18687**	18687	125.1	18746
UDS	S-S.	80	45	0	13765	11236	TL	15408	13488	TL		**15223**	15223	955.6	15263
UDS	S-S.	80	45	1	12374	11014	TL	14021	11367	TL		**13683**	13653	TL	13717
UDS	S-S.	80	45	2	12360	9911	TL	13987	10283	TL		**13370**	13370	1501.6	13674
UDS	S-S.	80	45	3	12266	9416	TL	13530	10825	TL		**13224**	13219	TL	13308
UDS	S-S.	80	45	4	11898	9579	TL	13380	10445	TL		**12792**	12782	TL	12916
UDS	S-S.	80	45	5	11840	9875	TL	13374	10041	TL		**13095**	13065	TL	13144
UDS	S-S.	80	45	6	12873	11339	TL	14569	11857	TL		**13754**	13750	TL	14008
UDS	S-S.	80	45	7	12477	10878	TL	13873	11184	TL		**13623**	13591	TL	13703
UDS	S-S.	80	45	8	11243	8003	TL	12878	10315	TL		**12124**	12120	TL	12254
UDS	S-S.	80	45	9	12982	11468	TL	14440	12565	TL		**14037**	14037	1180.4	14112
UDS	S-S.	80	60	0	9716	7543	TL	11269	6119	TL		**10151**	10141	TL	10289
UDS	S-S.	80	60	1	9104	7456	TL	11080	6085	TL		**9565**	9546	TL	9695
UDS	S-S.	80	60	2	10089	7880	TL	11542	4783	TL		**10311**	10286	TL	10406
UDS	S-S.	80	60	3	10235	7551	TL	11918	5991	TL		**10327**	10318	TL	10439
UDS	S-S.	80	60	4	9552	6968	TL	11093	6266	TL		**9569**	9541	TL	9668
UDS	S-S.	80	60	5	9479	6154	TL	10466	6368	TL		**9079**	9009	TL	9165
UDS	S-S.	80	60	6	10759	8360	TL	12441	7203	TL		**11046**	11035	TL	11353
UDS	S-S.	80	60	7	9813	8099	TL	11807	8366	TL		**10257**	10205	TL	10323
UDS	S-S.	80	60	8	9618	6722	TL	11544	8572	TL		**10141**	10040	TL	10196
UDS	S-S.	80	60	9	9608		TL	11038	4463	TL		10203	9971	TL	**10169**
UDS	S-S.	80	75	0	8283	6252	TL	9877	2995	TL		8717	8242	TL	**8377**
UDS	S-S.	80	75	1	7774	6199	TL	9705	2174	TL		**8275**	8142	TL	8291
UDS	S-S.	80	75	2	7896	1747	TL	9256		TL		8134	7547	TL	**7742**
UDS	S-S.	80	75	3	8982	6389	TL	10628	1922	TL		**8953**	8948	TL	9039
UDS	S-S.	80	75	4	7857	5605	TL	9463	2557	TL		8130	7705	TL	**7904**
UDS	S-S.	80	75	5	8419	4775	TL	9967	3267	TL		8781	8279	TL	**8422**
UDS	S-S.	80	75	6	7428	3300	TL	9185	979	TL		8041	7449	TL	**7607**
UDS	S-S.	80	75	7	7869	4661	TL	9248	4255	TL		8140	7639	TL	**7791**
UDS	S-S.	80	75	8	8332	6243	TL	10001	4238	TL		8566	8150	TL	**8300**
UDS	S-S.	80	75	9	7843	5362	TL	9837	2279	TL		8268	8050	TL	**8265**
UDS	S-S.	90	30	0	19410	19410	2585.7	21062	21044	TL		22577	22577	20.4	
UDS	S-S.	90	30	1	19020	19020	1447.7	20698	20685	TL		21856	21856	9.0	
UDS	S-S.	90	30	2	19491	19491	346.3	21183	21183	3183.1		22529	22529	2.6	
UDS	S-S.	90	30	3	20086	20086	432.0	21945	21932	TL		23326	23326	2.5	
UDS	S-S.	90	30	4	19108	19108	2587.6	20751	20751	3428.1		22542	22542	10.6	
UDS	S-S.	90	30	5	19633	19631	TL	21122	20893	TL		22992	22992	24.7	
UDS	S-S.	90	30	6	19717	19716	TL	21585	21515	TL		23395	23395	8.6	
UDS	S-S.	90	30	7	21090	21090	312.3	22917	22917	2071.5		24427	24427	12.8	
UDS	S-S.	90	30	8	20042	20042	512.6	21704	21700	TL		23334	23334	41.9	
UDS	S-S.	90	30	9	18784	18782	TL	20358	19518	TL		22203	22200	TL	
UDS	S-S.	90	45	0	13593	12126	TL	15327	12474	TL		14832	14830	TL	
UDS	S-S.	90	45	1	14236	11748	TL	15979	12897	TL		15591	15567	TL	
UDS	S-S.	90	45	2	13455		TL	15063	12101	TL		14629	14552	TL	
UDS	S-S.	90	45	3	14185	11169	TL	16038	11062	TL		15329	15315	TL	

Continued on next page

					Policy										
					optimal			**largest gap**				**traversal**			
Instance					BPC algorithm			BPC algorithm			ŽKS	BPC algorithm			ŽKS
distr.	type	$\lvert O\rvert$	Q	No.	UB	LB	time t	UB	LB	time t	UB	UB	LB	time t	UB
UDS	S-S.	90	45	4	12643	10699	TL	14186	10945	TL		13552	13548	TL	
UDS	S-S.	90	45	5	13824	11376	TL	15460	13167	TL		15189	15148	TL	
UDS	S-S.	90	45	6	13653	11955	TL	15318	12560	TL		15009	14973	TL	
UDS	S-S.	90	45	7	13970		TL	15656	11240	TL		15295	15262	TL	
UDS	S-S.	90	45	8	14129	10259	TL	15829	13254	TL		15133	15089	TL	
UDS	S-S.	90	45	9	14092	11736	TL	15838	11957	TL		14943	14943	1508.3	
UDS	S-S.	90	60	0	10842	7529	TL	12639	1499	TL		11219	11196	TL	
UDS	S-S.	90	60	1	10922	8851	TL	13018	7523	TL		11318	11254	TL	
UDS	S-S.	90	60	2	11122	7871	TL	13106	6748	TL		11737	11583	TL	
UDS	S-S.	90	60	3	11023	7580	TL	12669	8801	TL		11580	11480	TL	
UDS	S-S.	90	60	4	9881	5642	TL	11490	4935	TL		10369	9968	TL	
UDS	S-S.	90	60	5	11143	8447	TL	12984	4555	TL		11824	11761	TL	
UDS	S-S.	90	60	6	10510	8018	TL	12115	6163	TL		11162	10758	TL	
UDS	S-S.	90	60	7	10967	8225	TL	12884	7547	TL		11197	11108	TL	
UDS	S-S.	90	60	8	10658	7042	TL	12353	8381	TL		11202	11051	TL	
UDS	S-S.	90	60	9	11009	7357	TL	12735	6881	TL		11302	11242	TL	
UDS	S-S.	90	75	0	8653	4931	TL	9901	2574	TL		8640	8435	TL	
UDS	S-S.	90	75	1	8795	5220	TL	10393	5288	TL		9315	8806	TL	
UDS	S-S.	90	75	2	9299	6612	TL	11171	242	TL		9801	9586	TL	
UDS	S-S.	90	75	3	8721	2913	TL	10336	3840	TL		8710	8633	TL	
UDS	S-S.	90	75	4	9106	6200	TL	10713		TL		9185	8969	TL	
UDS	S-S.	90	75	5	8727	4343	TL	10520	905	TL		9142	8685	TL	
UDS	S-S.	90	75	6	9020	5879	TL	11111		TL		9325	9191	TL	
UDS	S-S.	90	75	7	9387		TL	11418	2856	TL		9873	9689	TL	
UDS	S-S.	90	75	8	9107	6736	TL	11043		TL		9231	9078	TL	
UDS	S-S.	90	75	9	8992	5621	TL	11122	940	TL		9294	9120	TL	
UDS	S-S.	100	30	0	20398	19997	TL	22174	22156	TL		24027	24027	126.1	24027
UDS	S-S.	100	30	1	23887	23887	373.3	25991	25991	3132.8		29005	29005	2.6	29005
UDS	S-S.	100	30	2	22093	22093	620.1	24146	24146	2936.5		26046	26046	2.9	26046
UDS	S-S.	100	30	3	21831	21831	807.0	23772	23772	1255.3		**25540**	25540	2.9	25810
UDS	S-S.	100	30	4	24854	24854	172.6	27046	27046	2413.7		29448	29448	2.9	29448
UDS	S-S.	100	30	5	21854	21854	TL	23757	23706	TL		25799	25799	178.5	25799
UDS	S-S.	100	30	6	21375	21375	3120.2	23424	22399	TL		**25119**	25119	75.0	25128
UDS	S-S.	100	30	7	22684	22684	TL	24959	24868	TL		26792	26792	21.4	26792
UDS	S-S.	100	30	8	22385	22385	2893.3	24589	24526	TL		**25845**	25845	88.4	25875
UDS	S-S.	100	30	9	21122	19087	TL	23005	22373	TL		**24422**	24389	TL	24423
UDS	S-S.	100	45	0	14163	11392	TL	15897	12099	TL		**15328**	15296	TL	15493
UDS	S-S.	100	45	1	15954	12886	TL	18027	14308	TL		**17482**	17475	TL	17631
UDS	S-S.	100	45	2	15153	12000	TL	17225	11652	TL		**16625**	16605	TL	16757
UDS	S-S.	100	45	3	16262	14341	TL	18351	14360	TL		**17722**	17712	TL	17915
UDS	S-S.	100	45	4	14682	12655	TL	16752	12954	TL		**15949**	15930	TL	16127
UDS	S-S.	100	45	5	15226	11607	TL	16945	10574	TL		**16323**	16308	TL	16434
UDS	S-S.	100	45	6	15279		TL	17169	13239	TL		**16538**	16533	TL	16612
UDS	S-S.	100	45	7	14659	11753	TL	16288	12251	TL		**15991**	15967	TL	16080
UDS	S-S.	100	45	8	14567	12835	TL	16597	10199	TL		**16152**	16145	TL	16306
UDS	S-S.	100	45	9	14607	12080	TL	16400	13453	TL		**15962**	15882	TL	16032
UDS	S-S.	100	60	0	11883	7995	TL	13389	3537	TL		12036	11607	TL	**11905**
UDS	S-S.	100	60	1	12327	9268	TL	14122	6362	TL		12778	12363	TL	**12751**
UDS	S-S.	100	60	2	11393	7581	TL	13425	3400	TL		**11638**	11630	TL	11910
UDS	S-S.	100	60	3	11396	7652	TL	12566	7174	TL		**11525**	11314	TL	11527
UDS	S-S.	100	60	4	12076	9706	TL	14351	7872	TL		**12739**	12572	TL	12769
UDS	S-S.	100	60	5	11750	8012	TL	13645	6268	TL		**12356**	12287	TL	12423
UDS	S-S.	100	60	6	11652	8687	TL	13577	8983	TL		**12370**	12226	TL	12430
UDS	S-S.	100	60	7	12449	9314	TL	14513	9577	TL		**12726**	12656	TL	12854
UDS	S-S.	100	60	8	12552		TL	14410	9000	TL		**13196**	13032	TL	13233
UDS	S-S.	100	60	9	11457	7478	TL	13308	449	TL		12109	11745	TL	**11977**
UDS	S-S.	100	75	0	9670	6235	TL	11619	3221	TL		**9581**	9516	TL	9727
UDS	S-S.	100	75	1	10368	7444	TL	13139	79	TL		10737	10266	TL	**10534**
UDS	S-S.	100	75	2	9994	5532	TL	11997		TL		10346	9893	TL	**10087**
UDS	S-S.	100	75	3	10331	6879	TL	12598	3908	TL		**10759**	10584	TL	10833
UDS	S-S.	100	75	4	10253	5962	TL	12166		TL		**10153**	9967	TL	10185
UDS	S-S.	100	75	5	11006	6164	TL	13312	2964	TL		**11473**	11328	TL	11483
UDS	S-S.	100	75	6	9725	4659	TL	11581	3334	TL		**9687**	9561	TL	9784
UDS	S-S.	100	75	7	9472	5225	TL	11035	767	TL		**9535**	9360	TL	9605
UDS	S-S.	100	75	8	9815	5227	TL	11501	3504	TL		10206	9657	TL	**9905**
UDS	S-S.	100	75	9	11014	8186	TL	13299	1416	TL		**11441**	11270	TL	11464

Appendix D. Detailed Results for Instances of Muter and Öncan

Table D.11: Comparison between our BPC algorithm and the algorithm of Muter and Öncan (MÖ) regarding the number of optimally solved instances.

Instances			#opt for different policies							
			traversal		midpoint		return		*Total*	
Q	$\|O\|$	*#inst*	BPC alg.	MÖ	BPC alg.	MÖ	BPC alg.	MÖ	BPC alg.	MÖ
24	20	10	10	10	10	10	10	10	30	30
	30	10	**8**	7	9	9	9	9	**26**	25
	40	10	8	8	**9**	7	9	**10**	26	25
	50	10	**5**	4	**8**	6	6	6	**19**	16
	60	10	**8**	6	6	6	**8**	7	**22**	19
	70	10	**5**	2	4	**5**	**4**	3	**13**	10
	80	10	**3**	1	1	**2**	3	**4**	7	7
	90	10	2	2	0	**3**	1	**3**	3	**8**
	100	10	0	**2**	0	**2**	0	**4**	0	**8**
Subtotal		90	**49**	42	47	**50**	50	**56**	146	**148**
36	20	10	**8**	6	8	**9**	8	8	**24**	23
	30	10	**6**	4	**9**	7	**9**	6	**24**	17
	40	10	**3**	0	**4**	1	**3**	0	**10**	1
	50	10	**2**	0	**3**	0	**1**	0	**6**	0
	60	10	0	0	0	0	**1**	0	1	0
	70–100	40	0	0	0	0	0	0	0	0
Subtotal		90	**19**	10	**24**	17	**22**	14	**65**	41
48	20	10	**5**	3	**7**	4	**8**	5	**20**	12
	30	10	0	0	**3**	0	**2**	0	**5**	0
	40	10	**2**	0	**3**	0	**5**	0	**10**	0
	50–100	60	0	0	0	0	0	0	0	0
Subtotal		90	**7**	3	**13**	4	**15**	5	**35**	12
Total		270	**75**	55	**84**	71	**87**	75	**246**	201

Table D.12: Detailed results for the instances of Muter and Öncan.

			Policy																	
			optimal			**traversal**			**largest gap**			**midpoint**			**return**			**composite**		
$\lvert O \rvert$	Q	No.	UB	LB	time t	UB	LB	time t	UB	LB	time t	UB	LB	time t	UB	LB	time t	UB	LB	time t
20	24	1	648.8	648.8	397.6	742.8	742.8	13.2	676.0	676.0	7.5	702.0	702.0	2.7	868.8	868.8	10.9	654.8	654.8	138.3
20	24	2	655.2	655.2	458.1	752.4	752.3	153.7	691.6	691.6	357.5	711.6	711.6	145.9	892.4	892.4	30.7	665.6	665.5	264.7
20	24	3	674.4	674.4	46.9	770.4	770.4	33.3	718.4	718.4	8.8	754.4	754.4	6.8	909.2	909.1	2.9	686.4	686.4	30.0
20	24	4	580.0	580.0	16.4	668.0	668.0	2.7	605.2	605.2	4.5	627.2	627.2	2.5	809.2	809.2	8.1	588.0	588.0	7.3
20	24	5	569.2	569.2	92.8	613.2	613.2	2.9	619.6	619.6	34.0	648.4	648.4	17.5	725.2	725.2	15.7	571.2	571.2	50.3
20	24	6	522.8	522.7	1609.8	564.8	564.8	142.5	549.6	549.5	288.1	561.6	561.6	28.7	677.6	677.6	153.8	525.6	525.5	627.5
20	24	7	552.4	552.4	36.4	628.4	628.4	21.6	571.6	571.6	17.6	583.6	583.6	3.2	757.2	757.2	11.6	559.6	559.6	78.7
20	24	8	560.0	560.0	126.5	644.0	644.0	18.5	574.0	574.0	7.9	582.0	582.0	4.0	828.0	828.0	3.3	563.2	563.2	34.4
20	24	9	680.0	680.0	28.8	770.4	770.4	1.7	719.6	719.6	14.9	737.6	737.6	4.6	929.2	929.2	0.6	685.2	685.2	7.6
20	24	10	500.8	500.7	1275.8	593.6	593.5	1711.5	528.8	528.7	576.6	544.8	544.8	280.7	700.8	700.7	345.8	500.8	500.8	140.2
30	24	1	921.2	916.0	*TL*	1017.2	1017.1	76.7	959.2	959.2	19.8	989.6	989.6	26.7	1241.2	1241.1	290.1	926.0	925.0	*TL*
30	24	2	994.4	994.3	605.2	1102.4	1102.3	102.5	1048.8	1048.8	5.1	1114.4	1114.4	94.6	1309.2	1309.1	148.4	999.2	999.1	125.9
30	24	3	734.0	734.0	407.6	820.8	820.8	60.1	775.2	775.1	478.5	791.2	791.1	188.3	1038.8	1038.7	43.0	741.2	741.2	139.2
30	24	4	832.0	821.4	*TL*	916.8	916.7	1222.9	864.0	863.9	2702.4	902.8	902.7	323.0	1083.6	1083.5	224.4	836.0	829.2	*TL*
30	24	5	828.4	825.4	*TL*	952.8	952.7	398.3	876.4	875.5	*TL*	896.4	896.3	2672.7	1098.4	1098.3	882.1	834.8	834.7	2583.2
30	24	6	849.6	840.7	*TL*	955.6	943.2	*TL*	875.6	875.5	1116.6	907.6	907.5	210.6	1129.2	1128.5	*TL*	864.4	853.6	*TL*
30	24	7	820.4	815.6	*TL*	912.4	912.3	3494.8	858.8	858.7	618.4	892.8	892.8	497.4	1101.2	1101.1	860.1	844.8	826.9	*TL*
30	24	8	860.0	859.9	3310.4	972.0	971.9	408.0	914.0	913.9	2573.3	958.0	958.0	389.3	1166.0	1165.9	199.3	862.0	861.9	613.2
30	24	9	862.8	848.0	*TL*	946.8	945.1	*TL*	891.2	887.3	*TL*	925.2	923.7	*TL*	1154.0	1154.0	1308.1	856.4	856.3	2940.8
30	24	10	758.4	758.3	211.9	858.8	858.8	65.4	796.8	796.8	47.8	813.6	813.5	16.2	1078.8	1078.8	65.8	767.6	767.6	258.5
40	24	1	1195.6	1187.3	*TL*	1326.8	1325.5	*TL*	1258.8	1258.7	2365.7	1316.8	1316.7	687.2	1588.8	1588.7	962.1	1210.4	1207.6	*TL*
40	24	2	1055.6	1055.5	*TL*	1156.0	1156.0	128.1	1127.2	1127.2	824.5	1158.0	1158.0	301.5	1434.0	1433.9	320.2	1061.6	1061.6	1155.1
40	24	3	1189.2	1189.2	1328.6	1313.2	1313.2	100.4	1273.6	1273.6	44.3	1311.6	1311.6	85.1	1584.4	1584.4	78.4	1200.4	1200.4	181.2
40	24	4	1074.8	1074.8	667.1	1199.6	1199.6	149.5	1154.0	1154.0	357.4	1194.8	1194.8	107.0	1409.6	1409.6	185.2	1082.0	1082.0	362.9
40	24	5	1088.4	1088.3	2241.1	1223.6	1223.6	126.4	1118.4	1118.4	176.3	1166.4	1166.4	149.7	1434.4	1434.4	53.1	1092.4	1092.3	454.5
40	24	6	1135.6	1135.5	2217.8	1299.6	1299.5	335.6	1200.4	1200.3	245.5	1240.4	1240.3	272.3	1524.4	1524.3	225.6	1141.6	1141.6	172.4
40	24	7	1128.4	1128.3	3281.4	1265.2	1265.2	59.0	1187.6	1187.5	305.7	1242.4	1242.3	1097.6	1498.0	1497.9	158.3	1132.4	1132.3	1279.5
40	24	8	1034.4	1034.3	838.7	1188.0	1187.9	1286.6	1079.6	1079.6	203.7	1114.4	1114.4	159.7	1458.0	1458.0	390.2	1041.6	1041.5	733.9
40	24	9	1002.8	992.3	*TL*	1181.2	1167.3	*TL*	1054.4	1047.6	*TL*	1091.6	1087.9	*TL*	1391.6	1390.0	*TL*	1022.4	1012.2	*TL*
40	24	10	1128.8	1112.0	*TL*	1266.8	1266.7	998.4	1182.8	1179.5	*TL*	1215.6	1215.5	1411.5	1501.6	1501.6	138.9	1124.8	1123.5	*TL*
50	24	1	1383.2	1381.5	*TL*	1581.2	1581.1	329.2	1451.2	1451.2	334.6	1484.0	1483.9	182.4	1938.4	1938.4	70.2	1400.4	1400.3	1649.1
50	24	2	1237.2	1221.0	*TL*	1361.6	1361.5	*TL*	1303.2	1298.4	*TL*	1366.4	1356.3	*TL*	1706.0	1702.8	*TL*	1242.4	1231.8	*TL*
50	24	3	1346.4	1345.6	*TL*	1515.2	1515.1	447.3	1436.0	1435.9	1385.1	1488.8	1488.7	976.0	1881.6	1881.5	954.9	1357.2	1357.2	1833.6
50	24	4	1405.2	1392.9	*TL*	1568.4	1568.3	2421.1	1493.2	1489.0	*TL*	1539.6	1539.5	1176.6	1906.4	1900.1	*TL*	1408.4	1401.7	*TL*
50	24	5	1454.4	1451.6	*TL*	1633.2	1628.0	*TL*	1554.8	1554.7	1501.8	1600.8	1600.8	510.0	1962.4	1962.3	1469.7	1469.6	1468.7	*TL*
50	24	6	1365.6	1359.0	*TL*	1520.4	1516.7	*TL*	1454.0	1453.9	3261.0	1506.0	1505.9	772.4	1793.6	1793.5	*TL*	1370.8	1369.5	*TL*
50	24	7	1179.6	1166.6	*TL*	1350.0	1332.2	*TL*	1232.4	1231.7	*TL*	1266.0	1263.4	*TL*	1616.0	1615.9	1736.1	1196.0	1183.2	*TL*
50	24	8	1204.8	1199.7	*TL*	1357.2	1353.3	*TL*	1268.4	1265.9	*TL*	1306.4	1306.4	1185.8	1684.8	1667.3	*TL*	1206.8	1206.6	*TL*
50	24	9	1222.0	1218.1	*TL*	1373.6	1373.5	847.1	1272.4	1272.3	1173.7	1308.4	1308.4	371.2	1636.8	1636.7	927.2	1230.0	1228.4	*TL*
50	24	10	1354.4	1346.6	*TL*	1506.4	1506.3	1299.9	1442.0	1441.9	1781.1	1472.8	1472.7	1267.7	1847.6	1847.5	777.6	1361.6	1359.3	*TL*
60	24	1	1490.4	1487.4	*TL*	1653.6	1653.6	929.6	1569.2	1569.2	1550.5	1617.2	1617.2	637.0	1940.8	1940.8	729.3	1504.4	1504.3	3340.5
60	24	2	1407.6	1401.8	*TL*	1550.4	1550.4	600.1	1463.2	1463.2	603.0	1516.4	1516.4	867.5	1929.2	1929.2	976.9	1413.2	1413.2	1543.0
60	24	3	1623.6	1583.7	*TL*	1791.6	1756.7	*TL*	1692.0	1673.1	*TL*	1746.0	1730.8	*TL*	2186.4	2146.6	*TL*	1629.2	1595.4	*TL*
60	24	4	1475.6	1475.2	*TL*	1635.2	1635.1	1114.9	1562.4	1562.4	1755.6	1625.2	1625.1	1310.5	1994.0	1994.0	521.8	1484.0	1484.0	502.9
60	24	5	1698.4	1691.9	*TL*	1877.6	1877.6	216.6	1777.6	1777.6	772.3	1815.6	1815.6	499.3	2328.4	2328.4	457.3	1705.6	1705.6	2367.3
60	24	6	1824.0	1818.2	*TL*	2007.6	2007.5	1614.9	1931.2	1931.2	1659.8	2011.2	2011.1	1498.7	2375.6	2375.5	1764.0	1827.2	1827.1	2062.2
60	24	7	1726.8	1713.4	*TL*	1958.4	1942.1	*TL*	1793.6	1789.7	*TL*	1817.6	1816.7	*TL*	2338.8	2337.6	*TL*	1739.6	1737.6	*TL*
60	24	8	1678.8	1671.0	*TL*	1840.8	1840.7	1222.4	1777.2	1770.0	*TL*	1846.0	1838.5	*TL*	2282.8	2282.7	3495.0	1690.0	1686.7	*TL*
60	24	9	1375.2	1371.1	*TL*	1504.8	1504.8	813.0	1454.0	1453.0	*TL*	1508.8	1508.3	*TL*	1861.2	1861.2	1174.1	1383.2	1378.7	*TL*
60	24	10	1655.2	1655.2	2099.1	1831.2	1831.2	503.1	1746.8	1746.8	2245.2	1824.0	1824.0	1456.6	2236.0	2235.9	1172.3	1661.2	1661.2	484.3
70	24	1	1864.8	1826.0	*TL*	2069.2	2045.4	*TL*	1936.0	1911.4	*TL*	1998.8	1986.4	*TL*	2473.6	2450.3	*TL*	1878.8	1842.0	*TL*
70	24	2	1912.0	1878.7	*TL*	2183.6	2175.7	*TL*	1984.0	1965.6	*TL*	2022.0	2007.0	*TL*	2648.4	2623.0	*TL*	1928.4	1911.6	*TL*
70	24	3	2041.6	2025.7	*TL*	2276.0	2273.5	*TL*	2139.6	2139.5	2383.8	2194.4	2194.4	2921.5	2782.8	2781.3	*TL*	2034.0	2034.0	2078.1
70	24	4	1860.8	1852.9	*TL*	2084.4	2084.3	1396.4	1960.4	1959.0	*TL*	2039.2	2037.8	*TL*	2492.8	2492.7	3494.6	1872.0	1869.1	*TL*
70	24	5	1724.0	1704.5	*TL*	1907.2	1907.2	1907.0	1804.4	1800.9	*TL*	1858.4	1857.2	*TL*	2320.8	2318.5	*TL*	1725.2	1720.3	*TL*
70	24	6	1857.6	1842.5	*TL*	2106.8	2091.4	*TL*	1944.8	1943.0	*TL*	2006.4	1999.9	*TL*	2475.2	2462.5	*TL*	1861.6	1851.4	*TL*
70	24	7	1812.8	1802.3	*TL*	1983.2	1983.1	1426.8	1914.4	1912.5	*TL*	1969.2	1969.1	3103.1	2422.0	2422.0	1676.9	1816.8	1813.0	*TL*
70	24	8	1772.8	1764.1	*TL*	1961.6	1961.6	1288.0	1854.8	1854.7	1551.1	1918.0	1918.0	1647.7	2431.2	2431.1	2523.1	1780.4	1780.3	*TL*
70	24	9	2054.0	1995.6	*TL*	2244.0	2243.9	2123.6	2095.6	2091.7	*TL*	2145.2	2145.2	885.3	2760.8	2760.7	1444.0	2033.2	2023.2	*TL*
70	24	10	1812.4	1779.4	*TL*	2012.8	1982.3	*TL*	1885.6	1867.5	*TL*	1952.8	1937.3	*TL*	2431.2	2423.2	*TL*	1813.6	1792.7	*TL*
80	24	1	1904.0	1890.0	*TL*	2176.0	2174.1	*TL*	2002.8	2002.7	2540.0	2070.8	2068.1	*TL*	2602.0	2601.9	3138.9	1951.6	1909.1	*TL*
80	24	2	2112.8	2077.8	*TL*	2380.0	2361.6	*TL*	2206.4	2190.4	*TL*	2262.4	2243.4	*TL*	2986.0	2955.4	*TL*	2131.6	2100.7	*TL*
80	24	3	2126.8	2063.3	*TL*	2273.6	2273.6	1284.7	2199.6	2196.8	*TL*	2257.6	2257.6	1679.2	2773.2	2773.1	2863.0	2069.6	2068.5	*TL*
80	24	4	2006.8	1999.4	*TL*	2222.8	2221.5	*TL*	2122.4	2118.2	*TL*	2194.0	2186.5	*TL*	2709.2	2708.4	*TL*	2018.8	2016.0	*TL*
80	24	5	2366.0	2304.9	*TL*	2594.8	2594.7	1305.2	2454.0	2449.0	*TL*	2538.8	2535.5	*TL*	3076.4	3076.3	2921.0	2337.2	2328.5	*TL*
80	24	6	2088.0	2056.7	*TL*	2266.4	2264.3	*TL*	2166.0	2164.9	*TL*	2242.8	2239.8	*TL*	2782.4	2779.5	*TL*	2077.6	2070.5	*TL*
80	24	7	2000.4	1974.9	*TL*	2218.4	2213.4	*TL*	2076.8	2068.9	*TL*	2126.0	2122.7	*TL*	2717.2	2686.6	*TL*	1992.8	1989.5	*TL*
80	24	8	2246.0	2214.6	*TL*	2491.2	2479.7	*TL*	2331.6	2319.1	*TL*	2392.8	2387.9	*TL*	3031.2	3025.8	*TL*	2255.2	2238.5	*TL*
80	24	9	1932.0	1873.4	*TL*	2081.2	2074.3	*TL*	1994.0	1988.3	*TL*	2092.0	2084.8	*TL*	2530.0	2526.7	*TL*	1906.8	1885.1	*TL*
80	24	10	2183.2	2095.9	*TL*	2344.8	2344.7	2048.2	2202.4	2200.3	*TL*	2269.6	2268.8	*TL*	2805.2	2802.3	*TL*	2156.4	2123.3	*TL*
90	24	1	2444.8	2364.1	*TL*	2611.2	2611.1	2776.3	2501.2	2493.9	*TL*	2587.2	2581.2	*TL*	3221.6	3220.1	*TL*	2441.2	2386.3	*TL*
90	24	2	2278.8	2252.9	*TL*	2556.0	2529.7	*TL*	2376.4	2346.1	*TL*	2457.6	2431.9	*TL*	3079.2	3059.0	*TL*	2304.8	2271.2	*TL*
90	24	3	2378.0	2365.4	*TL*	2670.4	2670.3	2839.3	2514.4	2504.7	*TL*	2575.6	2572.6	*TL*	3287.6	3276.4	*TL*	2397.2	2378.2	*TL*
90	24	4	2461.2	2369.4	*TL*	2672.8	2669.1	*TL*	2488.8	2483.0	*TL*	2572.0	2564.4	*TL*	3295.6	3292.6	*TL*	2394.8	2384.8	*TL*
90	24	5	2210.0	2184.1	*TL*	2484.4	2472.6	*TL*	2329.6	2314.5	*TL*	2443.2	2389.3	*TL*	3054.8	3019.9	*TL*	2235.2	2204.9	*TL*
90	24	6	2367.2	2292.0	*TL*	2551.2	2543.0	*TL*	2412.8	2404.2	*TL*	2505.6	2496.1	*TL*	3090.4	3090.4	3088.3	2319.6	2311.7	*TL*
90	24	7	2481.6	2437.2	*TL*	2716.0	2710.1	*TL*	2588.0	2586.2	*TL*	2677.6	2670.9	*TL*	3279.6	3262.9	*TL*	2464.0	2444.7	*TL*
90	24	8	2349.2	2306.1	*TL*	2578.0	2546.6	*TL*	2465.6	2427.5	*TL*	2536.0	2506.7	*TL*	3095.6	3049.2	*TL*	2366.8	2321.2	*TL*
90	24	9	2436.4	2396.5	*TL*	2682.4	2676.2	*TL*	2511.6	2506.2	*TL*	2575.6	2567.2	*TL*	3258.0	3243.8	*TL*	2430.0	2416.7	*TL*
90	24	10	2445.2	2367.5	*TL*	2634.0	2633.4	*TL*	2519.6	2518.8	*TL*	2619.2	2609.4	*TL*	3137.2	3132.7	*TL*	2384.4	2376.6	*TL*
100	24	1	2909.2	2817.7	*TL*	3119.2	3119.1	*TL*	3013.6	3000.4	*TL*	3104.0	3093.3	*TL*	3798.8	3792.9	*TL*	2845.2	2836.0	*TL*
100	24	2	2742.4	2669.2	*TL*	3087.2	3004.2	*TL*	2824.8	2809.4	*TL*	2932.8	2924.1	*TL*	3682.0	3668.7	*TL*	2762.4	2685.2	*TL*
100	24	3	2661.2	2614.2	*TL*	2925.2	2916.7	*TL*	2770.4	2763.8	*TL*	2894.8	2877.0	*TL*	3584.4	3568.7	*TL*	2642.4	2627.2	*TL*
100	24	4	2725.2	2547.5	*TL*	2840.0	2831.4	*TL*	2687.6	2677.9	*TL*	2845.2	2771.2	*TL*	3500.0	3494.0	*TL*	2635.6	2561.2	*TL*
100	24	5	2692.8	2630.1	*TL*	2934.4	2928.1	*TL*	2797.6	2789.3	*TL*	2903.6	2876.7	*TL*	3616.4	3583.1	*TL*	2702.0	2649.2	*TL*
100	24	6	2493.2	2470.8	*TL*	2785.6	2765.1	*TL*	2621.6	2599.4	*TL*	2703.6	2686.4	*TL*	3338.0	3329.1	*TL*	2532.0	2486.5	*TL*
100	24	7	2731.6	2501.7	*TL*	2839.6	2830.5	*TL*	2758.8	2744.3	*TL*	2879.6	2815.0	*TL*	3426.4	3420.8	*TL*	2668.0	2586.7	*TL*
100	24	8	2578.0	2467.7	*TL*	2830.4	2784.6	*TL*	2616.4	2603.2	*TL*	2713.6	2697.1	*TL*	3319.2	3286.5	*TL*	2562.8	2492.9	*TL*
100	24	9	2672.0	2591.2	*TL*	2887.2	2880.1	*TL*	2787.2	2733.3	*TL*	2848.8	2837.1	*TL*	3581.6	3578.9	*TL*	2692.4	2602.1	*TL*
100	24	10	2705.6	2581.2	*TL*	2903.2	2888.9	*TL*	2698.8	2686.2	*TL*	2822.0	2779.3	*TL*	3632.4	3593.5	*TL*	2639.6	2599.3	*TL*
20	36	1	482.0	481.9	1497.0	528.0	527.9	362.4	513.2	513.2	29.2	549.2	549.2	96.3	664.8	664.7	108.2	488.0	488.0	1535.7
20	36	2	491.2	491.1	2230.6	541.2	541.2	523.8	523.2	523.2	520.5	545.2	545.2	243.0	672.0	671.9	95.8	496.0	495.9	869.6
20	36	3	492.8	492.8	217.3	540.8	540.7	204.8	530.0	530.0	89.2	562.0	562.0	15.5	678.8	678.8	73.4	502.0	501.9	569.4
20	36	4	469.6	445.8	*TL*	520.0	506.4	*TL*	481.6	481.5	2191.3	511.6	509.5	*TL*	652.8	644.5	*TL*	474.0	458.6	*TL*

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$\|O\|$	Q	No.	Policy																	
			optimal			**traversal**			**largest gap**			**midpoint**			**return**			**composite**		
			UB	LB	time t	UB	LB	time t	UB	LB	time t	UB	LB	time t	UB	LB	time t	UB	LB	time t
20	36	5	451.2	451.2	2607.4	491.2	491.1	1257.1	499.2	499.2	960.9	531.2	531.2	580.4	592.4	592.4	459.5	452.4	452.3	1732.6
20	36	6	368.8	368.7	543.3	392.0	392.0	315.3	402.0	402.0	179.8	422.8	422.8	86.0	492.8	492.8	26.2	368.8	368.8	206.5
20	36	7	441.6	441.6	303.7	490.4	490.3	226.2	466.8	466.8	147.7	483.6	483.6	109.4	613.6	613.5	375.5	449.6	449.5	726.0
20	36	8	459.6	429.7	TL	502.8	483.8	TL	484.8	457.7	TL	492.8	475.6	TL	670.8	656.5	TL	466.8	438.6	TL
20	36	9	494.0	494.0	83.4	546.0	546.0	40.8	530.0	530.0	6.4	556.0	556.0	5.9	694.8	694.8	25.8	498.0	498.0	83.6
20	36	10	364.8	364.7	651.5	411.6	411.6	583.8	392.8	392.8	129.9	414.8	414.8	164.2	521.6	521.6	227.7	370.8	370.8	566.4
30	36	1	677.6	666.1	TL	723.6	723.5	1631.5	718.4	718.4	102.3	747.6	747.6	99.4	948.4	935.2	TL	690.8	672.1	TL
30	36	2	724.4	720.2	TL	760.4	760.4	260.3	793.6	793.5	645.2	846.4	846.4	373.6	969.6	969.5	890.3	726.4	725.9	TL
30	36	3	567.6	562.9	TL	610.4	609.3	TL	607.2	599.1	TL	625.6	620.0	TL	806.4	806.4	960.8	567.6	567.5	2870.2
30	36	4	591.2	587.2	TL	645.2	644.5	TL	629.2	629.1	907.0	667.2	667.1	838.0	799.2	799.2	427.1	593.2	592.2	TL
30	36	5	592.4	592.4	369.7	650.0	649.9	229.8	638.4	638.4	508.3	652.4	652.4	69.3	799.2	799.2	153.8	598.4	598.4	421.9
30	36	6	594.4	594.4	733.7	644.4	644.3	621.5	630.4	630.4	19.0	678.4	678.3	151.9	812.4	812.4	268.3	602.4	602.3	408.4
30	36	7	588.0	585.8	TL	637.2	633.4	TL	625.2	625.1	325.8	657.2	657.1	199.0	810.0	810.0	281.9	589.6	589.5	1307.4
30	36	8	636.4	636.4	256.7	701.6	701.5	541.3	691.6	691.6	342.6	733.6	733.6	63.2	867.6	867.6	6.1	645.6	645.6	938.4
30	36	9	610.0	607.2	TL	651.2	651.1	786.8	653.2	653.1	455.8	680.0	680.0	920.9	842.0	841.9	466.8	612.0	611.9	2050.1
30	36	10	580.4	563.9	TL	627.2	624.7	TL	616.4	616.3	3283.7	639.2	639.1	2964.8	825.2	825.1	2500.8	587.2	569.5	TL
40	36	1	935.6	875.0	TL	998.4	938.6	TL	974.4	949.6	TL	1051.6	1005.0	TL	1252.4	1197.3	TL	937.6	882.2	TL
40	36	2	793.2	771.4	TL	844.0	839.3	TL	860.0	853.9	TL	902.8	886.6	TL	1111.2	1085.1	TL	806.0	780.7	TL
40	36	3	912.4	882.1	TL	962.4	954.1	TL	988.8	970.8	TL	1013.6	1000.7	TL	1213.2	1199.1	TL	915.2	892.7	TL
40	36	4	796.0	788.5	TL	862.8	858.1	TL	862.0	855.8	TL	910.0	906.4	TL	1064.8	1060.8	TL	800.0	794.1	TL
40	36	5	815.6	802.8	TL	895.6	876.8	TL	857.6	852.4	TL	893.6	893.6	958.5	1095.6	1087.8	TL	821.6	811.2	TL
40	36	6	821.6	812.6	TL	892.8	892.7	2029.4	872.8	872.8	1187.2	906.8	906.8	1062.3	1113.6	1113.5	2582.0	819.6	819.0	TL
40	36	7	808.0	808.0	2793.8	896.8	896.7	743.7	870.8	870.8	1049.1	906.8	906.8	608.7	1104.8	1104.7	1381.8	814.8	814.2	TL
40	36	8	778.4	759.1	TL	863.6	843.8	TL	832.8	830.3	TL	865.6	858.5	TL	1101.6	1087.9	TL	790.4	766.9	TL
40	36	9	781.6	731.8	TL	875.2	821.7	TL	824.4	789.7	TL	856.4	832.7	TL	1083.6	1033.7	TL	794.8	746.1	TL
40	36	10	806.0	792.6	TL	878.8	878.7	2094.2	855.2	855.2	1136.8	897.2	897.2	554.0	1108.0	1107.9	809.9	804.8	802.7	TL
50	36	1	1030.4	1009.8	TL	1130.4	1119.9	TL	1089.2	1076.9	TL	1123.2	1121.1	TL	1466.0	1451.8	TL	1039.2	1022.6	TL
50	36	2	898.8	886.4	TL	966.8	954.6	TL	984.8	966.4	TL	1020.0	1019.0	TL	1254.8	1254.5	TL	910.0	897.2	TL
50	36	3	1021.6	980.6	TL	1101.6	1072.0	TL	1097.2	1068.2	TL	1143.2	1118.1	TL	1405.6	1375.7	TL	1013.6	989.3	TL
50	36	4	1000.4	988.9	TL	1084.4	1084.3	2330.3	1080.4	1080.4	2214.4	1134.4	1134.3	2625.2	1384.0	1383.9	TL	998.4	995.1	TL
50	36	5	1136.8	1039.9	TL	1131.2	1131.1	2436.5	1142.4	1142.3	3356.9	1189.6	1189.5	2160.9	1424.4	1424.3	2609.7	1055.2	1048.3	TL
50	36	6	1020.4	987.5	TL	1100.4	1067.4	TL	1098.4	1077.4	TL	1154.4	1127.8	TL	1361.2	1330.0	TL	1022.4	994.5	TL
50	36	7	876.0	849.1	TL	964.4	941.7	TL	940.4	923.6	TL	964.4	952.0	TL	1197.2	1195.0	TL	878.4	858.3	TL
50	36	8	906.4	870.0	TL	974.4	949.8	TL	963.2	932.2	TL	984.8	974.1	TL	1263.2	1225.8	TL	906.4	875.1	TL
50	36	9	904.0	883.7	TL	966.4	961.1	TL	944.8	944.8	2731.1	988.0	987.9	1678.7	1226.4	1220.9	TL	902.8	891.3	TL
50	36	10	1015.2	988.2	TL	1097.2	1073.6	TL	1102.4	1081.6	TL	1132.4	1119.7	TL	1403.2	1380.9	TL	1023.2	994.1	TL
60	36	1	1135.6	1092.7	TL	1220.8	1177.8	TL	1224.0	1178.1	TL	1272.8	1229.9	TL	1502.8	1450.3	TL	1148.8	1100.0	TL
60	36	2	1097.6	1033.2	TL	1169.6	1114.5	TL	1145.6	1108.8	TL	1178.0	1156.8	TL	1504.8	1452.6	TL	1083.2	1037.9	TL
60	36	3	1212.4	1138.0	TL	1296.8	1222.9	TL	1298.0	1242.9	TL	1348.4	1305.8	TL	1667.6	1579.5	TL	1216.4	1142.3	TL
60	36	4	1093.2	1061.0	TL	1178.0	1138.9	TL	1183.6	1152.1	TL	1245.6	1215.6	TL	1487.6	1457.4	TL	1101.6	1064.6	TL
60	36	5	1264.0	1224.2	TL	1336.4	1322.9	TL	1330.0	1323.4	TL	1374.0	1368.4	TL	1746.4	1715.0	TL	1280.0	1235.6	TL
60	36	6	1376.4	1319.7	TL	1456.0	1417.1	TL	1466.0	1446.2	TL	1532.8	1515.9	TL	1828.0	1764.3	TL	1380.8	1326.1	TL
60	36	7	1324.0	1221.7	TL	1338.8	1337.3	TL	1305.2	1302.9	TL	1328.0	1326.9	TL	1694.8	1692.2	TL	1252.8	1234.9	TL
60	36	8	1246.0	1206.8	TL	1327.6	1302.5	TL	1342.4	1308.1	TL	1401.2	1373.6	TL	1710.4	1665.9	TL	1242.4	1213.9	TL
60	36	9	1102.0	1015.2	TL	1100.0	1094.2	TL	1155.6	1095.2	TL	1156.8	1144.6	TL	1404.4	1404.4	2978.6	1071.2	1022.7	TL
60	36	10	1221.2	1202.8	TL	1317.6	1299.8	TL	1306.0	1292.8	TL	1373.2	1360.9	TL	1672.0	1647.6	TL	1225.2	1207.5	TL
70	36	1	1439.2	1310.8	TL	1458.0	1411.1	TL	1440.8	1409.2	TL	1517.2	1468.5	TL	1839.2	1781.3	TL	1362.8	1318.3	TL
70	36	2	1411.6	1355.1	TL	1568.4	1507.2	TL	1494.0	1439.7	TL	1530.0	1496.3	TL	1973.2	1910.6	TL	1422.4	1371.9	TL
70	36	3	1522.0	1434.5	TL	1584.8	1576.7	TL	1571.6	1557.5	TL	1677.6	1621.3	TL	2026.4	2009.0	TL	1536.0	1444.9	TL
70	36	4	1425.6	1333.2	TL	1535.6	1438.1	TL	1533.6	1435.6	TL	1524.8	1515.5	TL	1886.0	1814.8	TL	1431.2	1339.3	TL
70	36	5	1304.0	1254.9	TL	1416.8	1352.3	TL	1355.2	1348.2	TL	1416.0	1405.3	TL	1814.0	1732.2	TL	1298.8	1263.1	TL
70	36	6	1358.4	1332.6	TL	1560.8	1435.6	TL	1451.2	1429.4	TL	1496.4	1477.2	TL	1835.2	1803.3	TL	1452.8	1338.2	TL
70	36	7	1360.0	1299.7	TL	1387.6	1383.8	TL	1460.0	1408.1	TL	1470.0	1464.8	TL	1792.8	1782.2	TL	1410.0	1308.0	TL
70	36	8	1348.0	1287.9	TL	1441.2	1398.3	TL	1416.4	1392.3	TL	1467.2	1447.4	TL	1844.0	1794.3	TL	1353.2	1294.3	TL
70	36	9	1516.4	1364.2	TL	1592.4	1559.2	TL	1566.0	1535.0	TL	1629.2	1593.1	TL	2035.2	2002.3	TL	1486.0	1447.0	TL
70	36	10	1403.2	1277.9	TL	1447.6	1385.8	TL	1412.8	1378.1	TL	1479.2	1445.8	TL	1818.0	1770.2	TL	1366.8	1284.9	TL
80	36	1	1473.2	1357.4	TL	1660.0	1507.2	TL	1539.6	1480.3	TL	1591.2	1536.8	TL	1957.2	1912.3	TL	1419.6	1381.1	TL
80	36	2	1620.8	1410.3	TL	1678.8	1639.5	TL	1632.4	1604.0	TL	1698.4	1667.1	TL	2255.6	2146.5	TL	1632.8	1505.8	TL
80	36	3	1547.2	1482.6	TL	1645.6	1580.6	TL	1651.6	1605.6	TL	1723.6	1674.8	TL	2100.0	2028.3	TL	1536.8	1492.3	TL
80	36	4	1518.4	1455.0	TL	1610.0	1552.0	TL	1637.6	1574.2	TL	1678.4	1627.9	TL	2065.2	1998.3	TL	1517.2	1463.7	TL
80	36	5	1874.8	1506.0	TL	1925.6	1785.5	TL	1847.6	1789.2	TL	1880.4	1852.3	TL	2408.4	2251.0	TL	1692.0	1663.5	TL
80	36	6	1551.2	1484.4	TL	1643.6	1585.1	TL	1674.8	1598.3	TL	1740.0	1678.3	TL	2119.6	2038.2	TL	1560.8	1496.6	TL
80	36	7	1528.8	1416.0	TL	1648.4	1530.9	TL	1624.4	1512.8	TL	1687.6	1570.4	TL	2020.8	1945.6	TL	1451.2	1421.1	TL
80	36	8	1664.4	1441.2	TL	1779.6	1722.7	TL	1743.2	1716.4	TL	1791.2	1780.5	TL	2268.8	2205.5	TL	1650.0	1610.8	TL
80	36	9	1487.6	1323.8	TL	1494.4	1449.6	TL	1508.4	1469.1	TL	1591.6	1547.1	TL	1928.8	1862.2	TL	1493.6	1361.8	TL
80	36	10	1661.2	1414.9	TL	1699.6	1641.9	TL	1662.8	1626.2	TL	1743.6	1688.1	TL	2122.0	2060.0	TL	1588.8	1528.3	TL
90	36	1	1863.6	1372.9	TL	1910.0	1837.8	TL	1888.4	1846.3	TL	1980.4	1918.0	TL	2462.4	2360.8	TL	1780.4	1728.3	TL
90	36	2	1732.8	1493.6	TL	1868.4	1746.7	TL	1746.4	1718.0	TL	1892.8	1784.7	TL	2370.4	2234.6	TL	1732.0	1601.5	TL
90	36	3	1855.2	1523.4	TL	1952.4	1845.4	TL	1899.2	1840.6	TL	1966.8	1901.3	TL	2458.8	2386.1	TL	1803.2	1712.9	TL
90	36	4	1778.8	1613.3	TL	1964.0	1853.9	TL	1930.4	1844.8	TL	2022.4	1922.5	TL	2480.8	2405.0	TL	1803.2	1684.9	TL
90	36	5	1642.8	1393.7	TL	1828.8	1722.5	TL	1753.2	1711.4	TL	1845.6	1784.2	TL	2301.6	2208.2	TL	1725.2	1587.8	TL
90	36	6	1808.4	1582.7	TL	1906.4	1787.6	TL	1848.8	1789.7	TL	1912.8	1860.3	TL	2350.4	2266.4	TL	1814.4	1655.5	TL
90	36	7	1803.6	1488.0	TL	1940.8	1879.4	TL	1940.8	1895.1	TL	2024.0	1971.9	TL	2486.4	2378.3	TL	1877.2	1721.4	TL
90	36	8	1765.2	1387.0	TL	1880.0	1764.6	TL	1880.0	1773.9	TL	1964.4	1848.3	TL	2352.8	2219.0	TL	1758.8	1658.9	TL
90	36	9	1858.4	1552.9	TL	1903.6	1852.1	TL	1878.8	1825.7	TL	1933.6	1884.2	TL	2411.6	2357.1	TL	1852.4	1714.8	TL
90	36	10	1861.2	1678.1	TL	1904.4	1833.6	TL	1908.0	1842.4	TL	1984.0	1927.9	TL	2370.8	2287.3	TL	1861.2	1714.1	TL
100	36	1	2164.0	1610.3	TL	2260.8	2155.2	TL	2284.0	2177.1	TL	2378.8	2269.0	TL	2870.8	2743.9	TL	2126.4	1874.5	TL
100	36	2	2084.0	1572.9	TL	2138.8	2083.6	TL	2117.6	2075.1	TL	2270.8	2164.2	TL	2756.4	2681.0	TL	2082.0	1933.1	TL
100	36	3	2060.8	1668.3	TL	2105.2	2029.7	TL	2132.0	2032.3	TL	2191.2	2127.5	TL	2693.6	2608.2	TL	1973.2	1892.9	TL
100	36	4	2014.4	1837.5	TL	2094.0	1959.1	TL	2080.4	1952.1	TL	2174.0	2045.4	TL	2699.2	2540.3	TL	1974.0	1828.5	TL
100	36	5	2012.8	1666.9	TL	2155.2	2027.3	TL	2071.6	2035.0	TL	2239.2	2126.5	TL	2665.6	2595.6	TL	2014.0	1890.1	TL
100	36	6	1827.6	1715.4	TL	2068.8	1932.4	TL	1948.0	1901.3	TL	2058.4	1982.7	TL	2594.8	2422.4	TL	1922.0	1786.6	TL
100	36	7	2012.0	1562.9	TL	2041.6	1988.6	TL	2164.8	2025.0	TL	2159.6	2095.4	TL	2602.4	2517.7	TL	1929.6	1876.0	TL
100	36	8	1934.4	1379.3	TL	1998.0	1934.1	TL	1994.0	1914.6	TL	2059.2	2000.0	TL	2464.4	2402.7	TL	1932.0	1767.5	TL
100	36	9	2021.2	1421.5	TL	2082.8	2016.9	TL	2166.0	2030.5	TL	2152.0	2096.7	TL	2814.8	2629.8	TL	1959.2	1882.4	TL
100	36	10	1992.4	1616.9	TL	2118.4	1994.3	TL	2088.4	1967.2	TL	2151.2	2042.5	TL	2746.8	2590.7	TL	1950.8	1841.4	TL
20	48	1	387.6	385.7	TL	414.8	414.7	1498.2	424.8	424.8	170.9	459.6	459.6	733.1	543.6	543.6	338.5	389.6	389.6	3021.1
20	48	2	395.6	392.3	TL	425.6	425.5	2256.9	435.6	435.6	2099.4	455.6	455.6	713.8	553.6	553.5	552.6	397.6	397.5	2094.8
20	48	3	397.6	397.6	2179.9	423.6	423.6	175.4	441.6	441.6	215.8	469.6	469.6	161.2	543.6	543.6	89.8	397.6	397.5	708.9
20	48	4	401.6	357.5	TL	415.6	390.9	TL	406.8	396.1	TL	428.8	419.7	TL	528.8	528.7	2157.4	378.8	366.8	TL
20	48	5	358.8	358.8	2917.6	390.8	390.7	2034.0	422.8	420.7	TL	444.8	444.7	1458.8	482.8	482.8	821.2	358.8	358.7	1488.8
20	48	6	344.8	336.0	TL	368.0	361.8	TL	375.2	375.1	1919.0	398.8	398.7	1085.7	458.8	458.8	1286.8	348.8	339.3	TL
20	48	7	367.6	360.2	TL	406.8	393.5	TL	384.0	384.0	618.1	408.8	408.8	1161.7	506.8	506.7	1587.2	407.6	367.0	TL
20	48	8	393.6	342.1	TL	415.6	373.6	TL	417.6	374.4	TL	411.6	393.0	TL	553.6	530.9	TL	399.6	349.0	TL
20	48	9	393.6	393.6	366.7	419.6	419.5	27.0	442.8	442.7	223.3	467.6	467.5	157.0	561.6	561.6	25.0	395.6	395.6	359.9
20	48	10	349.6	310.6	TL	388.8	337.9	TL	363.2	350.1	TL	377.2	376.2	TL	459.2	457.7	TL	353.6	318.9	TL
30	48	1	606.0	541.8	TL	579.2	578.7	TL	617.2	617.1	824.9	649.2	649.1	1345.6	772.0	764.9	TL	614.0	546.8	TL

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			Policy																	
			optimal			**traversal**			**largest gap**			**midpoint**			**return**			**composite**		
$\|O\|$	Q	No.	UB	LB	time t	UB	LB	time t	UB	LB	time t	UB	LB	time t	UB	LB	time t	UB	LB	time t
30	48	2	632.4	587.1	TL	655.2	625.6	TL	701.2	667.8	TL	741.2	722.0	TL	867.2	806.4	TL	635.2	592.1	TL
30	48	3	500.0	457.9	TL	542.0	493.1	TL	533.2	499.4	TL	554.0	521.2	TL	695.2	669.5	TL	503.2	463.1	TL
30	48	4	496.0	475.8	TL	542.0	513.8	TL	540.0	527.7	TL	564.0	563.9	2563.4	660.0	660.0	1507.3	502.0	480.6	TL
30	48	5	506.0	481.4	TL	532.0	526.5	TL	546.0	532.9	TL	570.0	553.4	TL	670.0	664.8	TL	504.0	487.4	TL
30	48	6	501.2	481.9	TL	525.2	506.9	TL	542.0	531.6	TL	588.0	571.7	TL	682.0	678.6	TL	507.2	487.3	TL
30	48	7	498.0	470.3	TL	522.8	506.4	TL	560.0	530.5	TL	578.0	566.9	TL	712.8	678.1	TL	516.0	476.5	TL
30	48	8	506.8	491.3	TL	542.0	529.4	TL	558.0	551.0	TL	592.8	592.7	3211.5	696.8	696.7	1945.6	510.8	493.8	TL
30	48	9	514.8	488.1	TL	546.8	517.3	TL	564.0	543.9	TL	592.8	569.5	TL	734.0	699.0	TL	520.8	492.2	TL
30	48	10	486.0	456.0	TL	520.0	494.9	TL	538.8	511.6	TL	576.0	534.6	TL	702.8	675.5	TL	492.8	460.7	TL
40	48	1	746.0	692.0	TL	781.6	727.2	TL	822.8	770.0	TL	884.8	823.5	TL	1008.8	961.3	TL	749.6	696.6	TL
40	48	2	724.8	607.6	TL	645.2	645.1	1958.4	682.4	682.3	3078.2	716.4	714.7	TL	860.4	860.4	2752.9	622.4	615.2	TL
40	48	3	732.8	700.1	TL	762.4	744.1	TL	818.8	787.7	TL	846.8	818.5	TL	1003.6	970.5	TL	737.6	706.4	TL
40	48	4	721.6	618.7	TL	656.4	656.4	3377.3	686.4	686.4	2397.5	726.4	726.4	875.9	844.0	844.0	2047.5	635.2	621.0	TL
40	48	5	634.0	622.0	TL	796.4	662.3	TL	678.0	677.9	2681.6	716.0	716.0	896.7	857.2	857.2	2424.7	632.0	626.3	TL
40	48	6	736.4	674.5	TL	764.4	722.3	TL	741.2	738.0	TL	771.2	771.1	1867.8	927.2	927.1	2539.1	692.0	679.4	TL
40	48	7	714.8	666.5	TL	782.4	733.2	TL	775.6	742.8	TL	792.4	780.0	TL	990.4	920.2	TL	719.6	669.8	TL
40	48	8	601.6	596.3	TL	645.6	641.8	TL	665.6	665.6	3482.0	693.6	692.1	TL	865.6	865.5	1361.3	603.6	599.2	TL
40	48	9	605.2	582.1	TL	777.6	630.2	TL	655.2	630.3	TL	692.4	675.2	TL	855.2	830.3	TL	614.4	590.0	TL
40	48	10	724.4	659.8	TL	722.0	717.2	TL	762.8	740.6	TL	788.0	777.6	TL	980.4	928.2	TL	712.4	665.8	TL
50	48	1	842.0	805.7	TL	920.0	872.6	TL	915.6	879.1	TL	949.6	917.5	TL	1211.6	1163.8	TL	946.0	813.7	TL
50	48	2	726.8	705.9	TL	764.0	740.4	TL	794.0	781.4	TL	848.0	826.0	TL	1163.6	1002.4	TL	728.0	708.4	TL
50	48	3	843.6	778.7	TL	895.6	837.5	TL	928.8	863.6	TL	970.8	912.5	TL	1168.0	1100.7	TL	854.4	781.6	TL
50	48	4	821.2	794.1	TL	880.4	845.9	TL	912.4	886.3	TL	959.2	939.2	TL	1152.4	1124.7	TL	830.4	798.6	TL
50	48	5	956.0	829.0	TL	904.8	885.0	TL	958.0	929.4	TL	998.8	976.5	TL	1172.8	1149.1	TL	963.2	834.9	TL
50	48	6	844.8	789.1	TL	882.8	832.8	TL	932.0	873.4	TL	980.0	922.3	TL	1142.4	1070.3	TL	850.8	792.5	TL
50	48	7	701.6	676.4	TL	877.2	733.4	TL	779.6	746.8	TL	795.6	777.9	TL	996.4	967.5	TL	737.6	682.0	TL
50	48	8	765.2	689.8	TL	762.4	739.8	TL	831.2	757.8	TL	823.6	791.2	TL	1030.4	981.1	TL	826.0	692.8	TL
50	48	9	813.2	701.1	TL	875.2	743.2	TL	877.6	769.9	TL	818.8	811.3	TL	1123.2	983.7	TL	823.2	705.3	TL
50	48	10	838.0	789.9	TL	885.6	837.7	TL	934.0	884.4	TL	968.8	921.0	TL	1187.6	1122.6	TL	849.6	793.8	TL
60	48	1	960.4	848.1	TL	1007.2	894.7	TL	1052.4	935.8	TL	1098.4	985.1	TL	1283.2	1147.4	TL	958.4	850.8	TL
60	48	2	873.2	808.6	TL	978.8	859.0	TL	999.6	892.4	TL	1037.6	932.0	TL	1289.6	1153.1	TL	818.4	811.7	TL
60	48	3	1035.6	894.1	TL	993.2	950.5	TL	1148.0	995.6	TL	1099.6	1055.5	TL	1306.4	1253.9	TL	970.4	899.5	TL
60	48	4	942.4	853.2	TL	984.4	903.9	TL	1024.4	948.9	TL	1051.2	1007.3	TL	1280.4	1178.5	TL	930.4	854.1	TL
60	48	5	1082.0	960.0	TL	1134.4	1013.6	TL	1163.2	1059.4	TL	1222.8	1105.1	TL	1498.8	1357.6	TL	1074.4	965.1	TL
60	48	6	1101.2	1036.4	TL	1142.4	1089.6	TL	1222.4	1161.9	TL	1275.6	1221.5	TL	1477.2	1409.0	TL	1099.2	1040.9	TL
60	48	7	1058.4	988.6	TL	1125.2	1059.0	TL	1136.4	1074.9	TL	1156.4	1095.0	TL	1457.2	1372.5	TL	1063.2	999.0	TL
60	48	8	1064.4	946.8	TL	1134.0	998.3	TL	1179.2	1052.1	TL	1241.6	1114.2	TL	1348.0	1326.8	TL	1068.4	949.8	TL
60	48	9	906.8	789.9	TL	947.6	837.5	TL	982.8	872.5	TL	1036.8	921.2	TL	1270.4	1113.1	TL	909.6	792.2	TL
60	48	10	1042.0	946.1	TL	1118.8	1003.7	TL	1148.0	1044.1	TL	1200.8	1100.4	TL	1467.6	1322.2	TL	1048.0	948.9	TL
70	48	1	1165.2	989.8	TL	1246.4	1089.8	TL	1270.4	1131.4	TL	1332.4	1192.0	TL	1593.2	1427.5	TL	1182.8	1039.9	TL
70	48	2	1156.4	1051.3	TL	1282.8	1168.5	TL	1244.8	1168.4	TL	1292.8	1217.8	TL	1629.6	1526.4	TL	1157.2	1087.1	TL
70	48	3	1252.0	1135.9	TL	1292.0	1219.3	TL	1385.2	1260.8	TL	1354.0	1318.4	TL	1646.8	1608.9	TL	1253.2	1140.9	TL
70	48	4	1176.0	977.4	TL	1216.4	1106.9	TL	1272.4	1153.0	TL	1342.0	1226.7	TL	1590.0	1456.8	TL	1172.4	1053.3	TL
70	48	5	1140.4	949.4	TL	1206.8	1049.0	TL	1132.8	1085.7	TL	1237.6	1139.0	TL	1469.6	1386.3	TL	1150.4	998.8	TL
70	48	6	1185.6	1029.1	TL	1244.8	1109.3	TL	1284.8	1147.9	TL	1328.8	1191.9	TL	1611.6	1445.5	TL	1186.8	1056.4	TL
70	48	7	1132.0	1027.3	TL	1194.8	1077.9	TL	1263.2	1139.7	TL	1314.0	1190.0	TL	1580.0	1435.2	TL	1129.2	1030.5	TL
70	48	8	1158.4	1015.2	TL	1152.4	1084.2	TL	1265.2	1124.7	TL	1305.2	1170.6	TL	1498.8	1432.5	TL	1161.2	1019.0	TL
70	48	9	1299.6	1130.4	TL	1259.6	1214.0	TL	1300.8	1239.1	TL	1360.8	1294.4	TL	1669.6	1602.6	TL	1301.6	1139.6	TL
70	48	10	1147.2	986.7	TL	1122.0	1079.6	TL	1252.4	1115.8	TL	1321.2	1174.7	TL	1592.4	1418.7	TL	1147.2	1012.2	TL
80	48	1	1224.0	1020.1	TL	1357.6	1171.8	TL	1356.8	1196.9	TL	1410.0	1246.1	TL	1731.6	1536.8	TL	1167.2	1092.2	TL
80	48	2	1359.6	949.6	TL	1471.6	1282.2	TL	1378.4	1302.8	TL	1451.2	1366.7	TL	1854.0	1722.3	TL	1307.6	1195.9	TL
80	48	3	1304.4	1140.0	TL	1330.4	1238.6	TL	1408.4	1297.3	TL	1450.4	1360.0	TL	1742.0	1627.5	TL	1292.4	1178.9	TL
80	48	4	1348.8	1142.0	TL	1301.6	1208.0	TL	1364.4	1253.2	TL	1418.4	1302.1	TL	1724.8	1584.2	TL	1256.4	1144.2	TL
80	48	5	1514.0	1093.2	TL	1495.2	1368.4	TL	1544.4	1431.4	TL	1607.2	1489.0	TL	1958.4	1790.2	TL	1421.6	1264.5	TL
80	48	6	1290.8	936.1	TL	1343.6	1222.8	TL	1406.0	1274.4	TL	1500.0	1349.4	TL	1781.6	1618.3	TL	1282.0	1161.7	TL
80	48	7	1274.4	916.0	TL	1330.8	1191.6	TL	1361.6	1229.7	TL	1417.6	1281.4	TL	1746.0	1563.2	TL	1266.0	1126.5	TL
80	48	8	1380.4	1091.1	TL	1448.8	1325.6	TL	1506.0	1374.3	TL	1558.8	1433.3	TL	1918.4	1748.5	TL	1393.6	1257.8	TL
80	48	9	1223.6	920.5	TL	1297.2	1138.8	TL	1356.8	1195.4	TL	1433.6	1265.1	TL	1706.4	1496.6	TL	1236.4	1074.8	TL
80	48	10	1293.6	933.4	TL	1360.8	1247.4	TL	1405.6	1295.6	TL	1466.0	1355.1	TL	1774.8	1634.8	TL	1304.8	1187.6	TL
90	48	1	1539.2	758.9	TL	1572.4	1415.9	TL	1642.4	1480.3	TL	1699.2	1546.0	TL	2079.2	1881.2	TL	1511.2	1255.6	TL
90	48	2	1480.8	1097.2	TL	1565.6	1383.1	TL	1469.6	1403.1	TL	1549.6	1462.3	TL	1946.8	1804.5	TL	1419.6	1253.2	TL
90	48	3	1514.8	1115.7	TL	1511.2	1424.0	TL	1636.4	1478.9	TL	1672.4	1530.6	TL	2093.6	1894.2	TL	1508.0	1345.9	TL
90	48	4	1541.6	1365.8	TL	1582.0	1439.5	TL	1648.8	1482.9	TL	1681.6	1552.4	TL	1991.2	1916.7	TL	1514.8	1342.5	TL
90	48	5	1477.2	960.9	TL	1477.2	1321.4	TL	1517.6	1373.7	TL	1564.4	1439.0	TL	1917.2	1756.5	TL	1381.6	1248.4	TL
90	48	6	1473.6	1174.9	TL	1530.8	1376.2	TL	1597.6	1439.5	TL	1562.4	1504.8	TL	2010.8	1814.0	TL	1471.6	1313.5	TL
90	48	7	1567.2	910.9	TL	1609.6	1449.6	TL	1665.2	1515.3	TL	1738.4	1593.3	TL	2078.4	1891.3	TL	1537.2	1357.7	TL
90	48	8	1508.4	947.1	TL	1485.2	1372.2	TL	1545.2	1432.2	TL	1616.4	1497.5	TL	2025.6	1777.6	TL	1488.0	1309.8	TL
90	48	9	1605.2	940.5	TL	1578.8	1432.5	TL	1620.0	1471.3	TL	1672.0	1527.2	TL	2066.4	1882.9	TL	1492.0	1328.7	TL
90	48	10	1498.0	883.3	TL	1585.2	1411.6	TL	1636.4	1474.0	TL	1719.2	1552.1	TL	2034.0	1820.3	TL	1497.2	1345.9	TL
100	48	1	1834.0	766.9	TL	1833.6	1688.8	TL	1878.4	1762.0	TL	1999.6	1843.8	TL	2388.0	2203.8	TL	1750.4	1434.3	TL
100	48	2	1657.2	653.9	TL	1810.0	1589.6	TL	1867.6	1643.3	TL	1850.0	1712.6	TL	2274.4	2120.7	TL	1729.6	1229.5	TL
100	48	3	1710.4	1477.6	TL	1683.2	1556.9	TL	1844.4	1617.6	TL	1934.4	1700.7	TL	2238.8	2070.6	TL	1692.8	1473.8	TL
100	48	4	1710.0	837.4	TL	1690.8	1522.0	TL	1744.0	1581.2	TL	1808.0	1657.2	TL	2116.4	2029.3	TL	1608.0	1342.5	TL
100	48	5	1684.8	1030.9	TL	1753.2	1582.2	TL	1739.2	1652.9	TL	1921.2	1730.6	TL	2337.6	2076.6	TL	1682.8	1351.2	TL
100	48	6	1585.2	1411.2	TL	1665.6	1497.4	TL	1698.8	1538.0	TL	1791.2	1604.7	TL	2155.2	1938.8	TL	1581.6	1410.7	TL
100	48	7	1678.4	1467.0	TL	1642.8	1518.4	TL	1718.8	1606.7	TL	1805.2	1668.1	TL	2259.6	1995.4	TL	1674.4	1251.9	TL
100	48	8	1594.0	999.4	TL	1682.0	1499.3	TL	1662.0	1544.9	TL	1796.8	1626.9	TL	2026.4	1922.2	TL	1580.0	1376.6	TL
100	48	9	1698.8	1070.1	TL	1665.6	1546.4	TL	1734.4	1615.2	TL	1791.6	1682.6	TL	2239.2	2086.8	TL	1678.0	1381.1	TL
100	48	10	1618.0	1064.2	TL	1682.0	1539.9	TL	1733.6	1585.5	TL	1823.2	1658.1	TL	2266.4	2066.0	TL	1581.6	1367.9	TL