Exact Solution of Picker Routing Problems in Zoned Warehouses with Scattered Storage

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Abstract

We present two picker routing problems in a warehouse split into multiple disjoint zones with a picker assigned to each zone. Scattered storage is applied, meaning an article can be stored at several pick positions in one or more zones, and capacity restrictions limit the number of collected articles for each picker. Both problems seek picker tours that collect all requested articles, with each tour operating within one zone. While the multi-zone picker routing problem (MZPRP) minimizes the total length of all picker tours, the balanced multi-zone picker routing problem (BMZPRP) balances the tour lengths by minimizing the length of the longest picker tour. Both problems constitute a three-level optimization problem. While on the higher levels, zone assignment decisions and the selection of pick positions must be made, on the lower level, picker tours for each zone are determined.

To solve both problems, we use a network-flow model with covering constraints and varying objectives. This type of model was recently presented for the single picker routing problem with scattered storage, building on an extended state space of the dynamic-programming approach by Ratliff and Rosenthal. Our model contains a network for each zone, and demand-covering constraints across all zones ensure that the requested articles are collected. Computational experiments, including large instances with up to 200 articles, show that our solution approach efficiently solves the two problems within (milli-)seconds. An analysis of the number of zones demonstrates that in larger warehouse layouts, zoning reduces costs compared to single-zone picker routing.

Keywords: picker routing, scattered storage, warehousing, zoning

1. Introduction

Although, especially in e-commerce warehouses, automation is increasing, *picker routing* is still an indispensable component of warehouse operations and is likely to remain so (Schiffer *et al.*, 2022). Picker routing determines the order in which required articles are collected from their storage locations in the warehouse, so that the length of the resulting picker tour, starting and ending at a fixed depot, is minimized. There are multiple reasons why picker routing is still highly relevant and will remain important in the future. On the one hand, like human pickers, autonomous mobile robots also need a predefined route to collect the required articles (Bock *et al.*, 2024). On the other hand, manual order picking is advantageous for some of the typical features of e-commerce orders. Highly volatile demand and tight delivery schedules are characteristics of online retail (Boysen *et al.*, 2019). Discount campaigns or seasonal sales lead to short-term increases in both the number of orders and the volume of individual orders, and a fast-delivery standard requires fast processing of incoming orders. Especially the combination of those two features leads to highly variable workloads, most automated warehouse systems have difficulties to adapt to (Löffler *et al.*, 2022). However,

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manually operated warehouses are flexible and can more easily handle peaks in demand by scheduling additional pickers. Despite the feasibility of automating order picking, many companies opt for manual processes. Their rationale is that humans can respond more flexibly to unexpected changes, particularly when logical reasoning is required — something machines cannot yet do effectively (Grosse *et al.*, 2015).

In this paper, we examine picker routing in *zoned* warehouses with *scattered storage*, as the combination of zoning and scattered storage is widespread in warehouses for online retail (e.g., Weidinger, 2018; Boysen *et al.*, 2019; Schiffer *et al.*, 2022). This includes major global players, such as Amazon Europe and the fashion retailer Zalando (Boysen *et al.*, 2019). In warehouses with scattered storage, also called *mixed-shelves storage* (Weidinger, 2018), some or all articles can be picked from more than one pick position (=storage location) in the warehouse. The main advantage of a scattered storage strategy is "that items of demanded SKUs are found close by irrespective of the position within the warehouse [so that] the distance to be covered for order picking is reduced this way" (Weidinger, 2018, p. 139). However, a picker may need to visit multiple pick positions to collect a large quantity of a single article (Weidinger, 2018).

Zoning is a layout design that divides the warehouse into disjoint zones. A pick zone comprises pick positions typically arranged closely together, where a subset of all *stock keeping units* (SKUs) in the warehouse is stored. In this work, we define an SKU as an article stored at a specific pick position. At least one picker is assigned to each zone to pick the demanded articles exclusively within this zone. In the literature, a distinction is usually made between one (Jane, 2000) and several pickers per zone (Chen *et al.*, 2013). We examine the case with one picker per zone, eliminating the need to consider congestion effects. *Picker congestion* or *picker blocking* occurs when a picker is blocked by another picker from passing or accessing a pick position (Parikh and Meller, 2009). The benefits of zoned warehouses are numerous: The picker attains a high ratio of time spent extracting articles compared to time spent traveling between pick positions, due to the restricted size of the zone. This also results in enhanced familiarity with the SKUs within the zone, and the picking time of a specific order can be significantly reduced (Gu *et al.*, 2007). Compared to order picking systems where several pickers move through the whole warehouse, zone picking reduces traffic congestion (de Koster *et al.*, 2007). Nonetheless, there also come along disadvantages with zoning: If the required articles from different customer orders are picked simultaneously in different zones, they must then be consolidated again according to the original orders. This usually requires a separate downstream process. However, if picking takes place one after the other in the different zones, containers with the partially picked articles are transferred from zone to zone. This can lead to unnecessary waiting times for pickers who are waiting for their bin for the next picker tour (Gu *et al.*, 2007).

For both picker routing problems presented in this work, a zoned single-block warehouse with parallel aisles and a depot located in each zone is given, where each zone is assigned a picker with a predefined capacity limiting the number of articles to be picked in that zone. A set of articles must be collected from pick positions distributed throughout the warehouse, assuming each article is stored at one or more pick positions. Both problems seek simultaneous picker tours within the zones that start and end at the depots in each zone, such that all demanded articles are collected and the capacities of the pickers are respected. While the *multi-zone picker routing problem* (MZPRP) focuses on minimizing the total distance traveled across all picker tours, the *balanced multi-zone picker routing problem* (BMZPRP) aims to achieve a more equitable distribution by minimizing the length of the longest picker tour. Figure 1 shows an optimal solution for the MZPRP for an instance with ten articles, three zones, and a picker capacity of five articles. The depots are marked in gray, and encircled SKUs indicate that the articles are collected from these pick positions. According to the problem description, both problem variants can be seen as a three-level optimization problem. The decision at the highest level concerns the zone assignment and determines which required articles are picked in each zone. The second level deals with the scattered storage aspect by defining the subset of SKUs to be picked per zone. Last but not least, the picker tours are determined at the third level. Since the decisions at all three levels are interdependent, they must be made simultaneously to solve the problem optimally. To the best of our knowledge, no exact solution method addressing these two problems has been published yet, despite its high relevance in the industry.

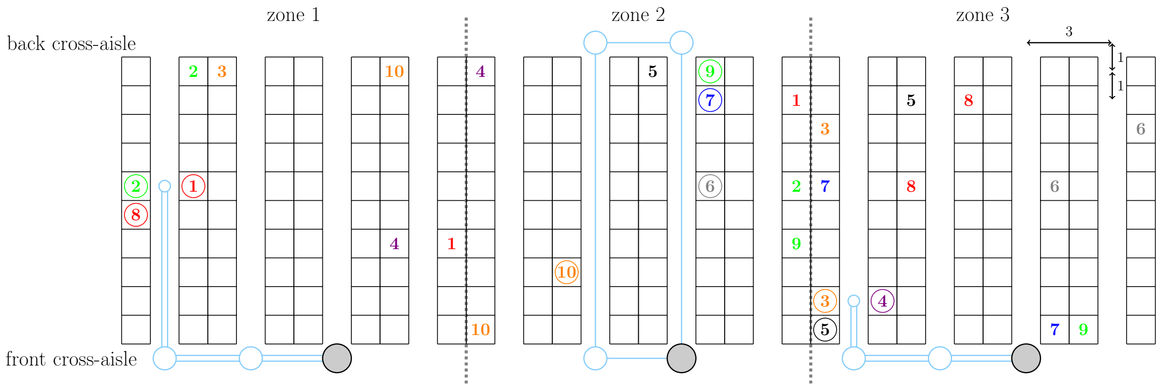


Figure 1: An optimal solution for an MZPRP instance with ten articles, three zones, and a capacity of $Q = 5$ for all pickers.

1.1. Contributions

This work contributes to further investigation of the two problem areas of zoning and picker routing in scattered warehouses, both of which have received little scientific attention to date. Despite their common occurrence in the industry, the combination of these challenges remains unexplored in the literature to the best of our knowledge. We define the two problems MZPRP and BMZPRP to address this research gap. The key contributions of this work are:

- We introduce two picker routing problems in zoned warehouses with scattered storage. While the MZPRP minimizes the total length of all picker tours, the BMZPRP minimizes the length of the largest picker tour to balance the picker tour lengths of all zones and, thus, the workload of all pickers. Both problems involve three levels of optimization, contributing to their computational complexity.
- We present an effective solution algorithm to both problems consisting of a network-flow model with different types of covering constraints that can be solved with the help of a *mixed-integer programming* (MIP) solver.
- We show that our generic model can also be applied universally to various routing problem variants of the (B)MZPRP that involve different warehouse layouts, multiple depots per zone, or routing strategies.
- Our extensive computational study, including very large instances with up to 200 required articles, shows that our solution approach efficiently solves both problems. Concerning the MZPRP, the vast majority of instances can be solved optimally within milliseconds. Only some of the largest instances with 200 articles require a few seconds. For the BMZPRP, the computation times are considerably longer for the largest instances with up to 60 aisles and 200 articles, and a few of these instances cannot be solved to optimality within the time limit.
- An analysis of the number of zones demonstrates that in larger warehouse layouts, implementing zoning can reduce costs by up to 12.6% compared to using a single-zone picker routing approach.

1.2. Structure

The remainder of this paper is structured as follows. In Section 2 we present a literature review on the topics of zoning and picker routing in scattered warehouses. Section 3 describes the single picker routing problem in warehouses with scattered storage and the extended state space of Heßler and Irnich (2024), which is the basis for our solution approach. Section 4 addresses the MZPRP and the BMZPRP and presents the new exact solution approach. Section 5 reports and analyzes the computational results. Final conclusions are drawn in Section 6.

2. Literature Review

The literature on zoning and picker routing in warehouses with scattered storage is very limited. However, zoning has a substantial impact on the picking performance (de Koster *et al.*, 2007) and scattered storage is widely used in practice (Weidinger, 2018). While only 6% of the surveyed literature by Gu *et al.* (2007) considered zoning, only 7% of the routing algorithms reviewed by Masae *et al.* (2020) referred to scattered storage. To the best of our knowledge, the combination of zoning and picker routing in warehouses with scattered storage has not yet been considered in the literature. Therefore, we will first focus on zoning and then on scattered storage.

Extensive surveys that take into account zoning are conducted by Gu *et al.* (2007), de Koster *et al.* (2007), Gu *et al.* (2010), Van Gils *et al.* (2018), and Boysen *et al.* (2019). In the literature, a distinction is usually made between two different types of zone picking. While simultaneous zone picking involves picking all articles in parallel in different zones, resulting in a high pick-rate of the pickers, sequential zone picking leads to a lower pick-rate, as the articles of an order are picked in one zone at a time (Parikh and Meller, 2008). However, simultaneous zone picking requires an additional process to sort the picked articles according to the original orders. This can either be done directly by the picker while picking (sort-while-pick), if the picking cart (or equivalent tool) has a sorting system, or in an additional downstream sorting process (pick-and-sort) (Choe, 1991). There are many different names for these two types of picking in the literature. Synonyms for simultaneous zone picking are concurrent (Gu *et al.*, 2007), parallel (Gu *et al.*, 2010), or synchronized (de Koster *et al.*, 2007) zone picking. Sequential zone picking is also called progressive zoning or pick-and-pass (de Koster *et al.*, 2007). In this work we consider simultaneous zone picking.

A batch is a set of orders where all the demanded articles are picked together, and batching describes the process of grouping customer orders into batches. Already Mellema and Smith (1988) showed that picking batched orders in zones is more productive than full-range picking of single orders. Batching is often combined with zoning for the picking process, as both operations are similar. For example Yu and De Koster (2009) described an approximation model that analyzes the effect of dynamic batching and zoning on the order throughput time for sequential zone picking. However, Parikh and Meller (2008) and Russell and Meller (2003) examined the batch versus zone problem where either batching or zoning is applied. Gray *et al.* (1992) used simulations to determine the optimal number of zones (as part of an integrated problem) in a fixed-size warehouse. The authors assumed that each zone consists of only one aisle and that several pickers can work in parallel in each zone, using simultaneous zone picking with a subsequent sorting process. de Koster *et al.* (2012) considered an almost identical warehouse setting, except that a zone can also consist of multiple aisles, and determined the optimal number of zones as an isolated problem. Petersen (2002) considered the effect of different batch sizes and shapes of the zones on the picker tour lengths. For a given number of zones and a given number of pick positions per zone, zone configurations differing in the number of aisles and pick positions per aisle are examined. The storage location assignment in a warehouse with sequential zone picking was examined heuristically by Jane (2000) with the goal of balancing the workload in the different zones. Jane and Laih (2005) concentrated on simultaneous zone picking and proposed a heuristic approach to the same problem, incorporating the co-appearance of distinct articles in the same order. For the objective function of minimizing the longest picker tour, an exact approach was proposed by Saylam *et al.* (2023) for simultaneous picking in warehouses with dynamic zones, i.e., zones can be reconfigured after each picking wave.

The *single picker routing problem with scattered storage* (SPRP-SS) aims to determine the shortest possible picker tour in a warehouse where each article to be collected by the picker may be stored at more than one pick position. If each article is stored only at a single pick position, there is no need to decide which subset of all required SKUs to visit, and the resulting problem is called *single picker routing problem* (SPRP). Ratliff and Rosenthal (1983) demonstrated that the SPRP is a well-solvable case of the NP-hard *traveling salesman problem* (TSP) (Gutin and Punnen, 2002). Using the dynamic-programming algorithm of Ratliff and Rosenthal (1983), this classic order picking problem can even be solved in linear time (Heßler and Irnich, 2022). However, the SPRP-SS is proven to be NP-hard in the strong sense by Weidinger (2018). Singh and van Oudheusden (1997) showed that the SPRP-SS is a special case of the traveling purchaser problem. Daniels *et al.* (1998) proposed a formulation based on the TSP and an efficient heuristic based on the tabu search metaheuristic for the SPRP-SS. Gu *et al.* (2007) asserted that picker routing with scattered storage is a barely investigated problem with a lot of research potential. Nevertheless, it took more than another ten years for the problem to be researched further. Weidinger (2018) proposed heuristics that select the pick positions for the required articles according to specific rules in a first step, before the dynamic program of Ratliff and Rosenthal (1983) for the resulting SPRP is then applied in a second step. Weidinger *et al.* (2019) added multiple depots to the SPRP-SS and provided heuristic solution methods. MIP-based solution approaches for parallel-aisle warehouses have been proposed by Goeke and Schneider (2021) and Su *et al.* (2023). While the former approach solves the SPRP-SS in single-block warehouses, the latter is designed for multi-block warehouses. Also for multi-block warehouses, Haouassi *et al.* (2025) provided an exact logic-based Benders decomposition method. The state-of-the-art exact approach is the one by Heßler and Irnich (2024). They extended the state space of Ratliff and Rosenthal (1983) which is the basis for a MIP model solved by a commercial solver. A more recent heuristic was presented by Wildt *et al.* (2025), who use transformation schemes for TSP variants to reduce the SPRP-SS to a TSP, which is then solved close to optimality by generic TSP solvers. The advantage of this method is that it is not limited to a specific warehouse layout. Instead of exact routing, Lüke *et al.* (2024) applied different rule-based routing strategies designed for the SPRP (traversal, midpoint, largest gap (Hall, 1993) and return, composite (Petersen, 1997)) to solve the SPRP-SS and showed that the NP-hardness of the problem remains in this case.

3. Single Picker Routing Problem with Scattered Storage

In this section, we define the SPRP-SS and briefly introduce the extended state space and modeling approach of Heßler and Irnich (2024). The MZPRP and the BMZPRP are then considered in the following Section 4.

3.1. Problem Definition

Given is a single-block warehouse with parallel aisles and a set of articles S. Each article is stored at one or more pick positions with a supply of ≥ 1 per pick position. The demand for each article $s \in S$ is denoted by $q_s \in \mathbb{N}$, whereby a distinction is made between two different cases: In the so-called *unit-demand* case, we assume that the articles of S always have a demand of one, so that only a single pick position per required article needs to be visited. In the *general-demand* case, the demanded quantity of an article may be greater than the supply at a specific pick position, meaning that several pick positions must be visited to satisfy the demand of one article. There is a single depot in the warehouse, which serves as the picker's start and end point. The picker aims to collect all the required articles in the warehouse in such a way that the resulting picker tour is as short as possible. We assume that the process of picking the articles from the pick positions does not require any additional distance.

3.2. Extended State Space and Modeling Approach of Heßler and Irnich (2024)

To be able to use Ratliff and Rosenthal's state space also for warehouse layouts with scattered storage, Heßler and Irnich (2024) extended the state space originally designed for the SPRP. For the sake of scope, we do not discuss the state space of Ratliff and Rosenthal (1983) in detail and refer the interested reader to their seminal paper. Instead, we describe the basic adaptions of the extended state space for the SPRP-SS.

The state space consists of nodes and (directed) edges. The nodes V of the extended state space remain the same compared to the nodes of the state space for the SPRP. Each node represents a combination of a specific *stage* and *state*. While a stage describes up to which aisle (numbered from left to right) the (still incomplete) picker tour has been constructed at this node, a state provides very limited information about the structure of the (incomplete) picker tour. Moreover, there is a designated origin node o and a destination node d in the state space.

With regard to the edges, we distinguish between the two edge types E^{aisle} and E^{cross}, which describe all possible traversing options (=actions) of the aisles and cross-aisles in a warehouse that can potentially result in an optimal solution. We assume $J = \{1, 2, \ldots, m\}$ denotes the set of aisles. Then the complete set of directed edges is defined as

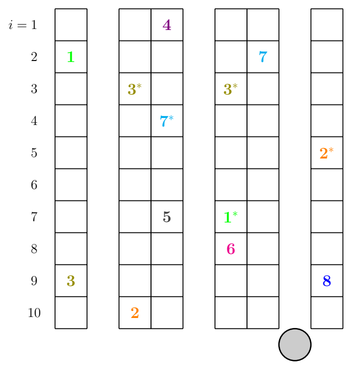
$$E = \bigcup_{j=1}^{m} \left(E_j^{aisle} \cup E_j^{cross}\right).$$

The cross-aisle actions E^{cross} are identical in the state spaces for the SPRP and the SPRP-SS and are not discussed in detail in this work. In contrast, the aisle actions E^{aisle} need some adaption, which we describe in the following.

Concerning the actions within an aisle, the six options

$$E^{aisle} = \{\texttt{1pass}, \texttt{2pass}, \texttt{top}, \texttt{bottom}, \texttt{gap}, \texttt{void}\}$$

only exist. `1pass` and `2pass` describe that an aisle is completely passed through once or twice, respectively. `top` (`bottom`) means that an aisle is entered from the back (front) cross-aisle and left again at the same cross-aisle after all required articles in the aisle have been collected. The action `gap` combines `top` and `bottom`, so an aisle is entered and exited from both cross-aisles while the largest part inside the aisle without any required articles (the so-called gap) is not visited. `void` indicates that an aisle is not visited at all. While the SPRP requires that all SKUs of the demanded articles are visited, as each article is stored at one pick position, the SPRP-SS necessitates only a subset of all SKUs to be visited to fulfill the demand for the required articles. As a result, for the SPRP-SS the actions `top`, `bottom`, and `gap` can be performed with different turning points in an aisle and are therefore no longer clearly defined. For this reason, an additional specification is required for each of these actions in order to indicate the turning point(s).



(a) An SPRP instance with extension into an SPRP-SS instance where the asterisks mark additional SKUs of the instance with scattered storage.

Type	E_1^{aisle} of SPRP	E_1^{aisle} of SPRP-SS
`1pass`	✓	✓
`2pass`	✓	✓
`top`(i)	Cell $i = 10$	$i \in \{2, 3, 9, 10\}$
`bottom`(i)	Cell $i = 2$	$i \in \{2, 3, 9, 10\}$
`gap`(h, i)	Cells $(h, i) = (2, 9)$	$(h, i) \in \{(2, 9), (2, 10), (3, 9), (3, 10)\}$
`void`	✗	✓

(b) Aisle actions of aisle 1 for the SPRP and the SPRP-SS in comparison. Actions marked in gray are dominated by cheaper actions.

Figure 2: Comparison of the aisle actions of a traditional state space (for SPRP) with the aisle actions of an extended state space resulting from the scattering of articles (for SPRP-SS).

Figure 2 demonstrates the differences between the aisle actions for the state space of the SPRP and the SPRP-SS using a small instance. The instances for both problem types are illustrated in Figure 2a. While the colored numbers without asterisks indicate the unique SKUs of the demanded articles for the SPRP, the numbers with asterisks represent additionally scattered SKUs of the same articles available only for the SPRP-SS. Figure 2b lists all actions for the first aisle for both problems. In the traditional state space for the SPRP exists exactly one edge for each possible action type. Thus, there is a maximum of six edges for each aisle. In the given SPRP instance, articles $1, 2$, and 3 need to be collected in aisle 1, so an edge of type `void` is not feasible in the first aisle, and we have a total of five edges to traverse aisle 1. Concerning the SPRP-SS, there are several pick positions where the demanded articles can be collected. We assume the unit-demand case, where the demand for each article is always one. This means that only a single pick position needs to be visited for each article. For example, article 4 needs to be collected from a specific pick position, while articles 1 and 3 can be picked from two and three different pick positions, respectively. As a result, the extended state space for the SPRP-SS has multiple edges of the traditional state space for the SPRP, and there can be several edges with different turning points of the same action type `top`, `bottom`, and `gap` within an aisle. Looking at Figure 2b, these edges are listed in the rightmost column for the first aisle. All three articles $1, 2$, and 3 in the first aisle can also be collected from different pick positions in other aisles, so it is not mandatory to pick any of these articles in aisle 1. This results, for example, in three `top` edges with turning points at pick positions $2, 3$, and 10, denoted `top`(2), `top`(3), and `top`(10), respectively, which allow picking a subset of all articles in aisle 1. Note that in the unit-demand case, an additional edge with turning point 9 is dominated by the cheaper action `top`(3). The graph for the SPRP, in contrast, has only one `top` action with a turning point at position 10. Parallel edges and domination also occur with the actions `bottom` and `gap`. Furthermore, for the SPRP-SS it is also feasible to collect none of the articles in aisle 1, which is realized with the action `void`. With general-demand instances, the state space often has fewer parallel edges. If the demand for an article can only be fulfilled if a specific pick position is visited (possibly in combination with other pick positions), only edges that represent aisle actions that visit this pick position are added to the state space for the corresponding aisle.

For the SPRP, an optimal picker tour is a shortest path in the state space of Ratliff and Rosenthal (1983) that can be computed with dynamic programming (DP) in linear time (Heßler and Irnich, 2022). However, the SPRP-SS cannot be solved by dynamic programming over the extended state space. The reason for this is the parallel edges of the same aisle action within an aisle (see Figure 2b), which no longer ensure that the demand for each required article is met. Therefore, a shortest-path problem is modeled based on the extended space with additional constraints that guarantee demand covering. The model is solved with a MIP solver.

Set E_s includes all edges $e \in E$ that contain pick positions of article $s \in S$. The supply of article $s \in S$ associated with edge $e \in E$ is denoted by b_{se}. Besides, the cost c_e represents the length of the (cross-)aisle action that is described by edge $e \in E$. Heßler and Irnich (2024) use binary variables x_e for all $e \in E$ that indicate whether edge e is part of the solution. They propose the following formulation to model the SPRP-SS:

$$z_{SPRP-SS} = \min \sum_{e \in E} c_e x_e \tag{1a}$$

$$\text{subject to} \quad \sum_{e \in \delta^+(\sigma)} x_e - \sum_{e \in \delta^-(\sigma)} x_e = \begin{cases} +1, & \text{if } \sigma = o \\ -1, & \text{if } \sigma = d \\ 0, & \text{otherwise} \end{cases} \qquad \forall \sigma \in V \tag{1b}$$

$$\sum_{e \in E_s} b_{se} x_e \geq q_s \qquad \forall s \in S \tag{1c}$$

$$x_e \in \{0, 1\} \qquad \forall e \in E \tag{1d}$$

The objective function (1a) minimizes the picker tour length. Flow conservation is guaranteed by constraints (1b). For each node $\sigma \in V$ the set of outgoing edges is denoted by $\delta^+(\sigma)$ and the set of ingoing edges is described with $\delta^-(\sigma)$. The flow-conservation constraints can also be written in a compact form as $\mathcal{N}\mathbf{x} = \mathbf{u}_o - \mathbf{u}_d$. In this case, $\mathcal{N}$ describes the incidence matrix of the state space (V, E), $\mathbf{x} = (x_e)_{e \in E}$ the vector of the x-variables, and $\mathbf{u}_o$, $\mathbf{u}_d \in \{0, 1\}^V$ the unit vectors of the origin and destination nodes of the state space. Constraints (1c) ensure that the demand is covered for all articles of the pick list. The domain of the x-variables is given by (1d).

4. Picker Routing in Zoned Warehouses with Scattered Storage

In this section, we first formally define the MZPRP and introduce some notation before we present our modeling approach for this problem. We then introduce the BMZPRP including necessary model adaptions. Finally, we point out that the presented model can also be used to solve problem variants of the (B)MZPRP without extensive adaptions.

4.1. Definition of the Multi-Zone Picker Routing Problem

We consider a single-block warehouse with parallel aisles, divided into a set of disjoint zones $\mathcal{Z}$ such that each aisle always belongs to exactly one zone (see Figure 1). Each zone has an assigned picker and is equipped with a depot located at the intersection of a cross-aisle and an aisle. The order pickers begin and end their picker tours at the depot of their zone. A given a set of different articles S with a demand q_s for each article $s \in S$ is to be collected in the warehouse. The articles are distributed throughout the warehouse, each stored at one or more pick positions $p \in P$. The set P^z contains all pick positions of zone $z \in \mathcal{Z}$ and P^z_s is a subset of P^z that only includes pick positions where article $s \in S$ is stored. b^z_{sp} indicates the supply of article $s \in S$ at position $p \in P^z_s$ in zone $z \in \mathcal{Z}$. Furthermore, each picker has a capacity Q of a maximum number of different articles that can be picked and transported during a tour. The objective of the MZPRP is to find picker tours within the zones with a minimum total length such that the demand for all articles is satisfied and the picker capacities are respected. We assume that each picker is equipped with the same type of picking cart, so that the capacities of the pickers are identical. In the case of individual picker capacities, the subsequent model can be easily adapted.

The special case of the MZPRP with only one zone is identical to the SPRP-SS. Since the SPRP-SS is proven to be NP-hard by Weidinger (2018), and the MZPRP is a generalization of the SPRP-SS, the MZPRP is therefore also NP-hard.

4.2. Network-Flow Formulation

We use the extended state space for the SPRP-SS (Heßler and Irnich, 2024) and combine it with the new aspect of zoning. More precisely, for each zone $z \in \mathcal{Z}$ a distinct extended state space $(V, E)^z$ is constructed in order to identify an optimal path within each state space that minimizes the total length across all paths. Each $o - d$ path in a state space represents a feasible picker routing tour in the corresponding zone in the sense that the tour starts and ends at the depot and only includes feasible (cross-)aisle actions. In addition, it must be ensured that the demand for all articles is satisfied and that the capacity of the pickers is not exceeded.

We denote by E^z_p the subset of edges E^z of zone $z \in \mathcal{Z}$ that includes pick position $p \in P^z$. Compared to the SPRP-SS, the cost of an edge is extended by the dimension of zones, i.e., c^z_e describes the length of the (cross-)aisle action that is depicted by edge $e \in E^z$ in zone $z \in \mathcal{Z}$. Accordingly, the binary variable x^z_e takes the value one if edge $e \in E^z$ of zone $z \in \mathcal{Z}$ is part of the picker tour, otherwise the value zero. Additional integer variables y^z_s for all $s \in S$ and $z \in \mathcal{Z}$ indicate the collected quantity of article s in zone z. In the case of unit demand, these y-variables are only binary. In this section, we formulate the model for the MZPRP. In the subsequent Section 4.3, the BMZPRP is described, and necessary adjustments to the formulation of the MZPRP are discussed. We propose the following network-flow formulation:

$$z_{MZPRP} = \min \sum_{z \in \mathcal{Z}} \sum_{e \in E^z} c_e^z x_e^z \tag{2a}$$

$$\text{subject to} \quad \mathcal{N}^z \mathbf{x}^z = \mathbf{u}_o^z - \mathbf{u}_d^z \qquad \forall z \in \mathcal{Z} \tag{2b}$$

$$\sum_{p \in P_s^z} \sum_{e \in E_p^z} b_{sp}^z x_e^z \geq y_s^z \qquad \forall s \in S, \forall z \in \mathcal{Z} \tag{2c}$$

$$\sum_{z \in \mathcal{Z}} y_s^z \geq q_s \qquad \forall s \in S \tag{2d}$$

$$\sum_{s \in S} y_s^z \leq Q \qquad \forall z \in \mathcal{Z} \tag{2e}$$

$$x_e^z \in \{0, 1\} \qquad \forall e \in E^z, \forall z \in \mathcal{Z} \tag{2f}$$

$$y_s^z \leq \sum_{p \in P_s^z} b_{sp}^z \qquad \forall s \in S, \forall z \in \mathcal{Z} \tag{2g}$$

$$y_s^z \in \mathbb{N}_0 \qquad \forall s \in S, \forall z \in \mathcal{Z} \tag{2h}$$

The objective function (2a) minimizes the total length of all picker tours within the zones. The flow-conservation constraints (2b) are built on those of the SPRP-SS (formulated in the compact form), except that they are extended by the zone dimension, as a separate state space is constructed for each zone $z \in \mathcal{Z}$. The collected quantity of an article within a zone must not exceed the available supply of this article in the zone, which is guaranteed by constraints (2c). Demand constraints (2d) make sure that the demand for each article is satisfied. Constraints (2e) ensure that the capacity of the picker is respected in each zone. The domains of the variables are defined by (2f) - (2h). Note that (2g) is optional and can be omitted.

The size of formulation (2) is summarized below: Let us denote the number of different articles by $a = |S|$ and the total number of pick positions where those articles are stored by n. The number of variables is bounded by $\mathcal{O}\left(m + n^2 + a \cdot |\mathcal{Z}|\right)$, which is composed as follows. For the SPRP-SS, the number of x-variables is bounded by $\mathcal{O}\left(m + n^2\right)$ (Heßler and Irnich, 2024), which remains the same for the MZPRP. Each state space has stages for the aisles of the zone where edges start and end, which together result in the summand m. The second summand n^2 relates to the parallel edges that emerge from scattered storage. In the worst case, all relevant pick positions are in a single aisle, which can lead to $\mathcal{O}\left(n^2\right)$ edges of type $\mathtt{gap}(h, i)$. This also remains independent of the zoning aspect. The number of y-variables in a zone coincides with the number of different articles stored in this zone. Thus, there is a total of $a \cdot |\mathcal{Z}|$ y-variables.

The number of constraints is bounded by $\mathcal{O}\left((m + a + 1) \cdot |\mathcal{Z}| + a\right)$. The flow-conservation constraints (2b) account for the addend $m \cdot |\mathcal{Z}|$, constraints (2c) and (2g) are constructed for each demanded article in each zone, contributing the summand $a \cdot |\mathcal{Z}|$. Moreover, constraints (2d) are required for each article s and constraints (2e) are built for each zone, i.e., a and $|\mathcal{Z}|$ constraints.

4.3. The Balanced Multi-Zone Picker Routing Problem

Minimizing the total length of all picker tours is the most common objective in the literature about warehousing (Weidinger *et al.*, 2019) and, thus, a reasoned objective for the MZPRP. As order picking costs can be responsible for more than half of the total warehouse operating expense (Tompkins *et al.*, 2010), shorter picker tours result in major savings. However, the route plan with minimal total length can lead to imbalanced picker tour lengths (Gu *et al.*, 2007). If the picker in one zone is allocated shorter tours on average than the pickers in the other zones, this leads to a lower work volume with unnecessary waiting times for that picker. This imbalance grows with the number of zones and reduces as the number of required articles increases (Gray *et al.*, 1992). Thus, we introduce the BMZPRP, which balances the picker tour lengths of all zones. Instead of minimizing the total picker tour length, we use a min-max objective that minimizes the length of the longest tour. This is known as the system throughput time for simultaneous zone picking (de Koster *et al.*, 2012). In the context of scheduling problems (Pinedo, 2022) and vehicle-routing problems (Irnich *et al.*, 2014) this kind of objective is also common.

To adapt the network-flow formulation, the objective function that minimizes the total length of all picker tours (2a) needs to be replaced by the following min-max objective function:

$$z_{BMZPRP} = \min \quad T \tag{3a}$$

The continuous variable T represents the length of the longest picker tour which is defined in additional constraints of the network-flow formulation as

$$\sum_{e \in E^z} c_e^z x_e^z \leq T \qquad \forall z \in \mathcal{Z} \tag{3b}$$

$$T \geq 0. \tag{3c}$$

The resulting model consists of (3a) - (3c) together with (2b) - (2h), describing the BZPRP with a workload-balancing objective. The number of variables is virtually identical to the MZPRP, only the new variable T is added. The number of constraints increases by additional $|\mathcal{Z}|$ constraints and is bounded by $\mathcal{O}\left((m + a + 2) \cdot |\mathcal{Z}| + a\right)$.

4.4. Universally Applicable Model

Although the model itself is fairly simple with demand, supply, and capacity constraints, the heart of the formulation is rather subtle with the state spaces for each zone. The fact that a separate state space is created for each zone reduces the problem to an SPRP-SS on the level of the zones with demand constraints on the global level of the complete warehouse including all zones. The main advantage of this approach is that the extended state space can be easily substituted by other state space concepts or adapted to other problem variants of the SPRP-SS. Therefore, our modeling approach can serve as a basis to describe other problem variants of the (B)MZPRP without the need for extensive adaptations.

Possible problem variants relate to the warehouse layout, the depot, or the routing (Heßler and Irnich, 2024, Table 1). Examples include multi-block warehouses or warehouse layouts with non-parallel aisles, modification of the depot(s) resulting in picker tours with different start and end points, and routing strategies other than exact routing. Notably, our modeling approach allows the (B)MZPRP to accommodate such alternative routing strategies by generating a thinned-out state space (Lüke *et al.*, 2024) for each zone.

5. Computational Study

In this section, we first address the instances of the computational study. Therefore, we describe the warehouse layout and the parameters used for the instances, and explain the generation of the instances. We then present and analyze the results for the two problems. First, we focus on the MZPRP and, in addition, analyze the impact of the number of zones. Next, we examine the BMZPRP and end with a comparison of the MZPRP and the BMZPRP.

The algorithms are implemented in C++ with the callable library of CPLEX 20.1.0 and compiled into 64-bit single-thread release code using Microsoft Visual Studio 2022. The MIP solver uses all CPLEX default parameters except the time limit and the setting to restrict the solver to a single thread. The computational experiments are conducted on a 64-bit machine running Microsoft Windows 10, equipped with an Intel® Core™ i7-5930K CPU at 3.5 GHz and 64 GB of RAM.

5.1. Instances

Since, to the best of our knowledge, neither the MZPRP nor the BMZPRP has yet been considered in the literature, there are no benchmark instances to date. Therefore, we have created a comprehensive set of instances for the unit-demand and general-demand cases which are used for both problems.

As already mentioned, we assume a one-block warehouse with parallel aisles and two cross-aisles at the back and the front of the aisles. The warehouse is divided into disjoint zones so that each aisle is assigned to exactly one zone, and each zone consists of adjacent aisles. Besides, all zones should have the same number of aisles, i.e., be the same size. If exact uniformity is not feasible, the zone sizes differ by no more than one aisle. The warehouse layout including the relevant distances is illustrated in Figure 1. The distance between two adjacent pick positions is one unit, as is the distance from the first or last position of an aisle to the nearest cross-aisle. Two adjacent aisles have a distance of three units. The number of pick positions per aisle is fixed to 80, whereby the pick positions are divided between two racks with 40 positions each on the left and right side of the aisle. We assume the articles stored at the pick positions to be rather small, so it is possible to store different articles at the same pick position. The aisles are wide enough to allow the picker to change direction during a traversal of the aisle (for the aisle actions `top`, `bottom`, or `gap`), but the picker can reach the pick positions on both sides of the aisle without an increase in travel distance. Roodbergen and Vis (2006) showed that a depot located in the middle of a cross-aisle in a single-block warehouse minimizes the average picker tour lengths. Therefore, in each zone, we place the depot at the intersection of the middle aisle and the front cross-aisle. If the zone has an even number of aisles, the depot is located in front of the aisle immediately to the right of the center aisle. We define the picker capacity as a function of the number of different required articles and the number of zones, namely as $Q = \left\lceil \frac{a}{|\mathcal{Z}|} \times 1.5 \right\rceil$. On the one hand, this approach ensures distributing the workload among multiple pickers. On the other hand, the capacity is sufficiently large to avoid implicitly constraining the solution by predefining the pick list size for each picker. The scatter factor is set to $\alpha = 2$, meaning each article is stored on average at two different pick positions in the warehouse. The articles are assigned randomly to the pick positions throughout the warehouse. In unit-demand instances, the demand for each article and the supply of each SKU are always one. In general-demand instances, the demand for each article is between one and five, with a decreasing distribution such that the demand is one (five) with a probability of 60% (5%). The supply of each SKU is uniformly distributed between one and three.

We vary several parameters to get different instance settings to allow detailed analysis of the results. As shown in Table 1, we generated instances with 10 to 60 aisles, one to five zones, and 10 to 200 articles to be picked. For each combination of parameter values, 50 instances were created. This results in a total of 5 000 different instances for both the unit-demand and the general-demand cases. The instances are available online at https://logistik.bwl.uni-mainz.de/research/benchmarks/.

Parameter	Value(s)
Number of aisles m	$10, 20, 35, 60$
Number of pick positions per aisle	$80\ (2 \times 40)$
Number of zones $\lvert\mathcal{Z}\rvert$	$1, 2, 3, 4, 5$
Number of articles a	$10, 25, 50, 100, 200$
Picker capacity Q	$\left\lceil \frac{a}{\lvert\mathcal{Z}\rvert} \times 1.5 \right\rceil$
Scatter factor α	2
Number of instances per setting	50

Table 1: Parameters used for the generation of the instances.

Due to the picker capacity in each zone, infeasible instances may arise, specifically when the number of articles to be collected exceeds the available capacity in a zone, and the articles are not sufficiently stored in other zones. To ensure the feasibility of the instances, we solved a *constraint satisfaction problem* in advance for each generated instance using the MIP solver. This problem consists of the demand constraints (2d), the capacity constraints (2e), and the constraints (2g) and (2h) to define the domain of the y-variables. If an instance was found that did not comply with these constraints, it was discarded and not considered in our study. However, this only occurred a few times, with ten different articles to collect.

5.2. Results for the Multi-Zone Picker Routing Problem

We first discuss the results of the MZPRP. Table 2 shows the average computation times in seconds for the MZPRP for the different parameter values of the number of aisles m in the warehouse, the number of zones (synonymous with the number of pickers) $|\mathcal{Z}|$, and the number of required articles a. Furthermore, we distinguish between the unit-demand case and the general-demand case. The time measurement includes the generation of the MIP model and its solution by the MIP solver. Values are rounded to two decimal places.

Note that each of the 10 000 instances can be solved optimally within few (milli-)seconds, which highlights the effectiveness of our MIP-based approach for solving the MZPRP. The number of zones does not seem to have a noticeable impact on the computation times. It is striking that the computation times for unit demand are many times longer than for general demand. The difference in computation time grows with an increasing number of articles. This is because, in the unit-demand case, only one pick position has to be frequented for each article. In contrast, in the general-demand case, several pick positions may need to be visited to satisfy the demand for an article. The necessity of visiting several pick positions for an article reduces the number of feasible aisle actions. This is the case if some pick positions of an article must be visited and are not optional to fulfill the demand of the article. As a result, fewer parallel edges correspond to aisle actions in the state space for the general-demand case than in the unit-demand case. The effect becomes particularly clear the more articles there are to collect. Since each edge in the state space corresponds to an x-variable in the MIP model, this relationship can also be observed when evaluating the number of variables of the models generated from the instances of the computational study. Table 3 shows the ratio of the average number of the x-variables of unit demand to general demand. Consistent with the reasoning just given, all values are ≥ 1, since unit demand results in at least as many parallel edges as general demand. It can also be noted that the values increase the more articles need to be collected. For the unit-demand instances with 200 articles, on average 2.41 times as many x-variables are generated as for the general-demand instances, which in turn explains the different computation times for these two cases. The number of the y-variables is identical in the unit-demand case and general-demand case and is therefore not considered.

Another observation that can be made from Table 2 is that for instances with many required articles, especially in the unit-demand case, the computation times first increase with an augmenting number of aisles but then decrease for warehouse layouts with 60 or already 35 aisles. A possible explanation for this phenomenon relates to the fact that the total number of edges is bounded by $\mathcal{O}(m + n^2)$. In the worst case, all articles are stored in only one aisle of the warehouse. As a result, the aisle actions of the type `gap` can lead to a high number of edges in the state space, which is quadratic in the number of relevant pick positions ($\mathcal{O}(n^2)$) (Heßler and Irnich, 2024). However, if the articles are distributed over many aisles, the effect becomes insignificant. In the unit-demand case, instances with up to 20 or 35 aisles and 200 articles have a high density of relevant pick positions per aisle and, therefore, a very high number of edges in the state spaces, leading to longer computation times. However, instances with only 10 aisles are solved relatively quickly within 2.5 seconds or less. We observed that the routes of instances with 10 aisles and 200 articles mainly consist of the aisle action `1pass` as many pick positions in the few aisles need to be visited. Perhaps these types of routes can be found easier by the solver than more complex routes. In the case of general demand, the effect is much weaker due to the smaller number of parallel edges. As the number of aisles increases, the relevant pick positions are distributed over more aisles, and the number of parallel edges in the state spaces for unit demand and general demand becomes increasingly similar. Again, this effect can also be seen in Table 3, where the number of variables of the unit-demand case in relation to the general-demand case is highest for 10 aisles and decreases with an augmenting number of aisles.

In the following, we analyze the average cost of the MZPRP. Table 4 has the same structure as Table 2 and presents the average optimal objective function values since all instances were solved optimally. Values are rounded to one decimal place. In general, the average cost rises as the number of articles or aisles increases. This is because more pick positions must be visited during the route, or the pick positions to be visited are spread over a larger area. It is also noticeable that the costs for the unit-demand case are, on average, lower than the costs for the general-demand case. This can be logically justified, as in the

m	$\|\mathcal{Z}\|$	unit demand $a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$	general demand $a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$
10	1	0.01	0.04	0.14	0.62	2.39	0.01	0.02	0.04	0.08	0.16
	2	0.02	0.05	0.15	0.79	2.39	0.01	0.02	0.04	0.08	0.19
	3	0.01	0.04	0.14	0.43	2.02	0.01	0.02	0.04	0.07	0.15
	4	0.01	0.03	0.08	0.20	0.51	0.01	0.02	0.03	0.05	0.11
	5	0.01	0.02	0.04	0.09	0.22	0.01	0.01	0.02	0.03	0.06
20	1	0.01	0.04	0.15	0.69	12.98	0.01	0.02	0.05	0.10	0.18
	2	0.02	0.05	0.15	0.85	13.37	0.02	0.03	0.05	0.10	0.21
	3	0.02	0.06	0.17	0.87	8.81	0.02	0.03	0.06	0.10	0.20
	4	0.02	0.07	0.18	0.79	11.65	0.01	0.03	0.05	0.10	0.19
	5	0.02	0.06	0.19	0.81	6.80	0.01	0.03	0.05	0.08	0.17
35	1	0.02	0.03	0.08	0.45	10.87	0.02	0.03	0.04	0.11	0.23
	2	0.03	0.05	0.10	0.48	10.25	0.02	0.03	0.05	0.10	0.23
	3	0.03	0.06	0.13	0.57	14.78	0.03	0.04	0.05	0.11	0.24
	4	0.03	0.06	0.15	0.65	14.54	0.02	0.03	0.06	0.11	0.23
	5	0.04	0.11	0.15	0.68	16.08	0.02	0.04	0.07	0.10	0.20
60	1	0.03	0.05	0.09	0.21	4.31	0.03	0.03	0.05	0.08	0.24
	2	0.04	0.06	0.10	0.25	5.26	0.03	0.04	0.05	0.09	0.21
	3	0.04	0.07	0.09	0.31	6.09	0.04	0.04	0.06	0.09	0.24
	4	0.05	0.06	0.11	0.38	10.62	0.03	0.05	0.06	0.10	0.22
	5	0.06	0.09	0.16	0.50	9.86	0.05	0.05	0.07	0.10	0.21

Table 2: Average computation time in seconds for the MZPRP.

m	$\|\mathcal{Z}\|$	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$
10	1	1.11	1.36	1.67	2.46	3.56
	2	1.11	1.35	1.64	2.48	3.55
	3	1.11	1.34	1.69	2.42	3.80
	4	1.13	1.36	1.65	2.29	3.73
	5	1.09	1.36	1.59	2.43	3.73
20	1	1.05	1.15	1.31	1.74	2.53
	2	1.06	1.16	1.31	1.76	2.55
	3	1.06	1.14	1.30	1.70	2.63
	4	1.05	1.16	1.31	1.75	2.58
	5	1.04	1.16	1.30	1.74	2.59
35	1	1.03	1.08	1.16	1.40	1.88
	2	1.03	1.08	1.17	1.39	1.88
	3	1.03	1.08	1.15	1.38	1.90
	4	1.03	1.09	1.18	1.40	1.91
	5	1.03	1.08	1.16	1.40	1.92
60	1	1.02	1.04	1.08	1.21	1.50
	2	1.02	1.05	1.09	1.22	1.50
	3	1.02	1.04	1.08	1.21	1.50
	4	1.02	1.04	1.09	1.20	1.49
	5	1.02	1.04	1.08	1.20	1.50
Total		1.05	1.16	1.30	1.69	2. 41

Table 3: Number of the x-variables generated from the MZPRP instances of the unit-demand case in relation to the general-demand case.

unit-demand case the demand for an article is always one, and the picker, therefore, only has to visit a single pick position for each article. In the case of general demand, on the contrary, the supply at a pick position may not be sufficient to fully satisfy the demand for an article, which might require the picker to visit several pick positions to satisfy the demand for an article.

		unit demand					general demand				
m	$\lvert\mathcal{Z}\rvert$	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$
10	1	156.2	235.2	301.9	357.3	403.3	189.0	289.3	364.0	429.7	460.8
	2	165.5	239.1	299.8	352.9	398.9	205.0	291.8	363.6	426.4	466.4
	3	181.5	254.7	302.6	353.7	399.4	217.7	310.2	368.2	428.2	469.8
	4	191.4	266.5	315.7	364.7	398.5	236.0	325.0	386.0	425.1	465.6
	5	205.4	285.5	329.1	376.4	412.8	237.8	338.1	403.0	435.3	440.0
20	1	218.3	344.3	468.2	596.1	711.5	260.6	421.9	574.9	732.1	853.6
	2	219.4	341.6	462.9	589.7	704.9	265.4	417.7	568.6	728.2	857.1
	3	225.7	354.4	467.9	584.4	694.6	276.9	429.5	563.0	722.6	852.7
	4	230.7	354.3	466.8	576.9	694.2	283.0	445.2	570.2	719.8	846.7
	5	241.6	375.5	474.1	587.0	695.6	284.4	448.1	581.4	726.9	854.6
35	1	284.6	463.3	649.5	862.6	1083.0	339.8	558.0	788.5	1061.7	1326.6
	2	278.1	445.8	643.3	848.6	1066.2	324.8	558.2	767.1	1049.2	1339.2
	3	278.2	449.7	633.3	850.8	1070.6	342.9	547.0	777.0	1039.6	1317.0
	4	281.2	450.1	635.0	840.7	1059.0	328.6	563.4	769.3	1044.0	1314.5
	5	283.7	450.1	636.0	840.4	1059.2	328.7	564.9	765.0	1042.3	1324.8
60	1	389.0	619.4	875.2	1192.4	1571.2	451.6	726.6	1025.5	1449.4	1924.4
	2	366.6	590.7	849.3	1184.7	1555.1	420.8	712.2	1011.0	1429.6	1916.6
	3	344.4	580.1	840.5	1176.6	1548.1	414.6	707.0	1018.0	1447.6	1922.9
	4	340.0	574.0	832.2	1162.6	1542.5	406.4	708.2	1002.7	1430.6	1923.4
	5	342.4	578.8	827.3	1151.6	1536.3	399.2	702.0	1005.6	1424.9	1930.9

Table 4: Average cost of the MZPRP.

We use the data from Table 5 to analyze the impact of the number of zones on the total length of the picker routes. The table shows the percentage difference in average cost between the SPRP-SS (=MZPRP with one zone) and the MZPRP with multiple zones. For each parameter combination of the number of aisles and articles, the average percentage cost difference of the instances with two to five zones is shown compared to the single-zone instances. Thus, negative values mean that the MZPRP has lower costs than the SPRP-SS or that zoning reduces costs. Note that the instance setting resulting in the lowest total length is written in bold. Table 5 reveals that in most settings, cost savings in the single-digit percentage range can be achieved by using two or more zones. This represents great potential, as order picking accounts for the majority of warehouse operating costs (Bartholdi and Hackman, 2019), and therefore, even small savings for individual routes add up to significant savings overall. Small instances with no more than 25 articles and 20 aisles usually have the lowest costs with only one zone. On the contrary, zoning can achieve by far the greatest savings for instances with 60 aisles and 10 articles. More precisely, in the unit-demand case (general-demand case), picking in four (five) zones can save 12.6% (11.6%) of the cost compared to the case in which the warehouse is a single zone.

Overall, in both the unit-demand case and the general-demand case, a higher number of zones is beneficial as the number of aisles increases. The same is true for an increasing number of articles, although the effect is diminished in the case of general demand for instance settings with a large number of aisles because of the very small percentage cost difference. In general, the average percentage cost differences due to the number of zones are slightly lower in the general-demand case than in the unit-demand case. In the general-demand case, visiting multiple pick positions per article may be necessary, limiting the options for collecting sufficient quantities. In contrast, the unit-demand case always requires only one pick position per article, providing more picking options and allowing greater cost savings by introducing more zones.

m	$\|\mathcal{Z}\|$	unit demand					general demand				
		$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$
10	1	**0.0%**	**0.0%**	0.0%	0.0%	0.0%	**0.0%**	**0.0%**	0.0%	0.0%	0.0%
	2	6.0%	1.7%	**-0.7%**	**-1.2%**	-1.1%	8.4%	0.9%	**-0.1%**	-0.8%	1.2%
	3	16.2%	8.3%	0.2%	-1.0%	-1.0%	15.2%	7.2%	1.2%	-0.4%	2.0%
	4	22.6%	13.3%	4.6%	2.1%	**-1.2%**	24.9%	12.3%	6.1%	**-1.1%**	1.1%
	5	31.5%	21.4%	9.0%	5.3%	2.4%	25.8%	16.9%	10.7%	1.3%	**-4.5%**
20	1	**0.0%**	0.0%	0.0%	0.0%	0.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%
	2	0.5%	**-0.8%**	**-1.1%**	-1.1%	-0.9%	1.9%	**-1.0%**	-1.1%	-0.5%	0.4%
	3	3.4%	2.9%	-0.1%	-2.0%	-2.4%	6.3%	1.8%	**-2.1%**	-1.3%	-0.1%
	4	5.7%	2.9%	-0.3%	**-3.2%**	**-2.4%**	8.6%	5.5%	-0.8%	**-1.7%**	**-0.8%**
	5	10.7%	9.1%	1.3%	-1.5%	-2.2%	9.2%	6.2%	1.1%	-0.7%	0.1%
35	1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	2	**-2.3%**	**-3.8%**	-1.0%	-1.6%	-1.5%	**-4.4%**	0.0%	-2.7%	-1.2%	0.9%
	3	-2.3%	-2.9%	**-2.5%**	-1.4%	-1.1%	0.9%	**-2.0%**	-1.5%	**-2.1%**	-0.7%
	4	-1.2%	-2.9%	-2.2%	-2.5%	**-2.2%**	-3.3%	1.0%	-2.4%	-1.7%	**-0.9%**
	5	-0.3%	-2.9%	-2.1%	**-2.6%**	-2.2%	-3.3%	1.2%	**-3.0%**	-1.8%	-0.1%
60	1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	2	-5.8%	-4.6%	-3.0%	-0.6%	-1.0%	-6.8%	-2.0%	-1.4%	-1.4%	**-0.4%**
	3	-11.5%	-6.4%	-4.0%	-1.3%	-1.5%	-8.2%	-2.7%	-0.7%	-0.1%	-0.1%
	4	**-12.6%**	**-7.3%**	-4.9%	-2.5%	-1.8%	-10.0%	-2.5%	**-2.2%**	-1.3%	-0.1%
	5	-12.0%	-6.6%	**-5.5%**	**-3.4%**	**-2.2%**	**-11.6%**	**-3.4%**	-1.9%	**-1.7%**	0.3%

Table 5: Percentage cost difference between the SPRP-SS and the MZPRP.

5.3. Results for the Balanced Multi-Zone Picker Routing Problem

In this section, we consider the BMZPRP with the objective of minimizing the length of the largest picker tour, which was introduced in Section 4.3. Table 6 shows the average computation time in seconds for the BMZPRP. The structure of the table is the same as that of the previous tables. The time measurement again includes the generation of the MIP model and its solution by the MIP solver. In contrast to the MZPRP, some instances with 200 articles cannot be solved optimally within the time limit of 600s. The number of affected instances is noted in the column *tl* (=time limit), and these instances are included in the average computation time with 600s.

To some extent, the required computation time to solve the BMZPRP behaves the same as the MZPRP. This includes the fact that general-demand instances can be solved significantly faster than unit-demand instances and that the computation times increase with the number of articles. But there are also differences compared to the MZPRP concerning the computation time. In particular, for instance settings with many articles and aisles, the computation time heavily increases with a growing number of zones, which is not the case for the MZPRP. As mentioned earlier, minimizing the length of the largest route does not change the number of variables and constraints in the MIP model much compared to minimizing the total length. Therefore, it seems that the objective function of the BMZPRP (3a) in combination with the additional constraints (3b) and (3c) makes the MIP significantly more difficult to solve than the MIP of the MZPRP.

Table 7 shows the average length of the largest picker route. Instances that were not solved to optimality within the time limit of 600s are listed in the column *tl* and are not included in the calculation of the average cost. As expected, the route lengths increase as both the number of articles and the number of aisles increase. In addition, the average length of the largest picker route decreases the more zones the warehouse has. For the same reasons as for the MZPRP, the routes in the general-demand case are at least as long as in the unit-demand case.

		unit demand						general demand				
m	$\|\mathcal{Z}\|$	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$	tl	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$
10	1	0.01	0.04	0.14	0.61	2.43		0.01	0.02	0.04	0.07	0.15
	2	0.04	0.10	0.40	1.73	3.54		0.02	0.03	0.07	0.10	0.24
	3	0.02	0.06	0.19	0.54	1.60		0.02	0.02	0.04	0.08	0.19
	4	0.02	0.05	0.08	0.19	0.72		0.01	0.02	0.03	0.07	0.13
	5	0.01	0.03	0.06	0.12	0.25		0.01	0.01	0.02	0.04	0.07
20	1	0.01	0.04	0.14	0.72	13.24		0.01	0.02	0.05	0.11	0.18
	2	0.09	0.16	0.42	2.36	40.11		0.04	0.06	0.12	0.21	0.30
	3	0.11	0.21	0.49	2.70	24.70		0.04	0.07	0.15	0.19	0.29
	4	0.07	0.22	0.47	4.49	112.73		0.04	0.09	0.11	0.23	0.27
	5	0.08	0.15	0.42	1.74	106.82	(4)	0.03	0.05	0.10	0.20	0.22
35	1	0.02	0.04	0.08	0.46	12.95		0.02	0.02	0.04	0.11	0.22
	2	0.14	0.21	0.42	1.63	38.17		0.06	0.09	0.14	0.26	0.54
	3	0.21	0.28	0.62	3.77	66.54		0.09	0.12	0.26	0.52	0.65
	4	0.25	0.38	0.83	9.19	122.13	(2)	0.09	0.18	0.30	0.75	0.86
	5	0.20	0.36	1.32	10.14	269.95	(7)	0.10	0.20	0.33	0.72	0.94
60	1	0.03	0.05	0.08	0.21	4.86		0.03	0.03	0.05	0.08	0.25
	2	0.15	0.27	0.37	1.10	21.50		0.10	0.12	0.19	0.32	0.65
	3	0.17	0.40	0.84	3.03	57.21		0.10	0.23	0.41	0.81	1.81
	4	0.24	0.60	1.82	9.04	144.09	(1)	0.12	0.27	0.66	1.75	4.29
	5	0.27	0.74	3.93	21.13	273.10	(4)	0.13	0.29	0.87	2.73	4.42

Table 6: Average computation time in seconds for the BMZPRP.

		unit demand						general demand				
m	$\|\mathcal{Z}\|$	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$	tl	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$
10	1	156.2	235.2	301.9	357.3	403.3		189.0	289.3	364.0	429.7	460.8
	2	97.0	127.7	158.4	186.2	206.0		113.2	157.2	193.6	221.7	238.4
	3	85.2	98.8	113.1	129.8	147.6		94.5	120.8	139.2	169.4	181.4
	4	75.4	88.4	93.0	105.4	117.8		85.0	99.3	118.3	133.0	149.6
	5	69.6	85.8	88.0	88.0	88.0		78.2	87.6	88.0	88.0	88.0
20	1	218.3	344.3	468.2	596.1	711.5		260.6	421.9	574.9	732.1	853.6
	2	124.8	179.7	241.5	303.1	359.5		144.5	223.5	293.4	376.6	437.8
	3	98.6	131.6	169.0	208.4	244.6		113.2	164.0	206.3	262.8	306.2
	4	85.4	108.4	130.4	157.0	187.0		98.6	133.4	163.6	199.5	225.6
	5	79.4	96.9	110.1	129.8	152.7	(4)	88.6	114.3	136.0	166.0	181.8
35	1	284.6	463.3	649.5	862.6	1083.0		339.8	558.0	788.5	1061.7	1326.6
	2	154.9	233.6	330.8	432.1	542.5		178.0	294.6	400.5	537.6	684.8
	3	117.8	165.0	223.1	295.0	367.2		138.9	200.4	277.8	364.6	458.8
	4	99.3	134.0	171.8	224.6	277.7	(2)	110.3	164.3	215.6	288.6	352.3
	5	87.3	113.4	143.1	182.0	225.3	(7)	99.9	136.8	175.7	229.5	288.0
60	1	389.0	619.4	875.2	1192.4	1571.2		451.6	726.6	1025.5	1449.4	1924.4
	2	203.7	312.4	436.8	600.5	783.3		234.6	376.7	516.7	725.8	969.6
	3	140.4	212.4	294.3	402.8	525.8		169.1	257.4	358.5	500.2	657.3
	4	115.4	165.6	224.4	304.7	396.3	(1)	137.0	203.7	275.9	376.4	501.4
	5	101.0	138.8	182.4	244.8	320.6	(4)	118.0	168.4	223.6	310.0	409.9

Table 7: Average cost of the largest route of the BMZPRP.

5.4. Comparison of the Two Problems

Table 8 compares the average total costs of the two problems. The values show the ratio of the average costs of the BMZPRP to the MZPRP.

Overall, it is noticeable that balanced route lengths increase the total length only moderately compared to minimizing the total length. The instance setting with 10 aisles, five zones, and 25 articles has the highest average cost increase at 29%, while the average cost increase across all instances is only 7%. In general, a warehouse layout with several zones increases the total length of the BMZPRP compared to the MZPRP. A possible explanation is that a higher number of tours in the warehouse potentially leads to higher additional costs if the objective is to minimize only the largest picker tour and all other tours are disregarded. However, in a warehouse with only one or two zones, there is no or only one additional route to the optimized route that can contribute to additional costs.

In addition, the total length of the BMZPRP is proportionally lower when the number of articles is large. This is because, with many articles, the single routes of the MZPRP become larger and more balanced, and therefore the additional cost for fully balanced routes is lower. This effect can also be observed in a weaker form when comparing unit demand with general demand. On average larger picker routes in the general-demand case lead to slightly lower additional costs than in the unit-demand case.

		unit demand					general demand				
m	$\|\mathcal{Z}\|$	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$	$a = 10$	$a = 25$	$a = 50$	$a = 100$	$a = 200$
10	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	1.10	1.04	1.04	1.04	1.02	1.05	1.04	1.03	1.02	1.01
	3	1.18	1.10	1.06	1.06	1.08	1.15	1.07	1.06	1.09	1.04
	4	1.21	1.21	1.13	1.04	1.02	1.16	1.11	1.06	1.08	1.10
	5	1.21	1.29	1.22	1.12	1.04	1.19	1.14	1.05	1.00	1.00
20	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	1.09	1.03	1.03	1.02	1.02	1.05	1.05	1.02	1.02	1.01
	3	1.18	1.06	1.06	1.05	1.04	1.12	1.08	1.07	1.06	1.05
	4	1.23	1.13	1.07	1.06	1.06	1.23	1.11	1.09	1.07	1.03
	5	1.27	1.21	1.09	1.06	1.07	1.25	1.16	1.10	1.09	1.04
35	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	1.07	1.03	1.02	1.01	1.01	1.04	1.04	1.03	1.02	1.02
	3	1.17	1.07	1.04	1.03	1.02	1.10	1.06	1.05	1.04	1.04
	4	1.25	1.12	1.06	1.05	1.04	1.20	1.10	1.08	1.08	1.05
	5	1.26	1.16	1.08	1.06	1.05	1.26	1.13	1.10	1.07	1.06
60	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	1.08	1.04	1.02	1.01	1.01	1.06	1.04	1.02	1.01	1.01
	3	1.12	1.06	1.04	1.02	1.02	1.13	1.06	1.05	1.03	1.02
	4	1.19	1.11	1.06	1.04	1.02	1.19	1.10	1.07	1.04	1.03
	5	1.23	1.12	1.07	1.05	1.03	1.23	1.13	1.08	1.07	1.05

Table 8: The average total length of the BMZPRP compared to that of the MZPRP, excluding instances not solved within the time limit.

6. Conclusions and Outlook

This work presented the multi-zone picker routing problem (MZPRP) and the balanced multi-zone picker routing problem (BMZPRP). Both optimization problems address picker routing in zoned warehouses with scattered storage and consist of three levels. The top level involves specifying in which zone(s) each demanded article is picked. The second level addresses the scattered storage issue and determines the SKU subset to be picked within each zone. Finally, the lowest level optimizes a picker tour in each zone. Since the decisions across all three levels are interrelated, they must be made simultaneously to achieve an optimal solution. While the MZPRP seeks minimum-length picker tours, the BMZPRP balances the picker tour lengths by minimizing the length of the largest picker tour.

To the best of our knowledge, no exact solution approach for both problems has been published so far, despite the significant relevance in the industry, where major global companies combine zoning and scattered storage with picker routing. We introduced a new solution algorithm to solve both problems exactly. The MIP-based approach uses the extended state space of Heßler and Irnich (2024) for picker routing in warehouses with scattered storage, which is based on the seminal dynamic program of Ratliff and Rosenthal (1983). Finally, a network-flow formulation is obtained and solved using a MIP solver.

An extensive computational study with a warehouse layout of up to 60 aisles and up to 200 required articles showed that the MIP-based approach is highly efficient for solving both problems. For the MZPRP, all instances are solved to optimality within milliseconds or a few seconds, depending on whether multiple pick positions have to be visited to satisfy the demand for an article (general-demand case) or the supply of an article at a pick position is guaranteed to be sufficient to fulfill the demand (unit-demand case). Moreover, our study revealed that zone picking can save up to 12.6% of the costs compared to single-zone picking. For the BMZPRP, the computation times are longer for the large instance settings with 200 articles. A few instances cannot be solved optimally within the time limit of 600 seconds. Even though the total length is not directly optimized in this problem, it is, on average, only 7% higher than for the MZPRP.

Our introduced solution approach is universally applicable since the extended state space used to build the network can be easily substituted by other state space concepts. Therefore, our mathematical formulation can serve as a basis for problem variants of the (B)MZPRP without major modifications. Future research could benefit from this feature and adapt the solution approach to possible problem variants, such as multi-block warehouses, depot modifications (Heßler and Irnich, 2024, Table 1), or heuristic routing strategies (Lüke *et al.*, 2024). Besides, the MZPRP or BMZPRP could also be integrated into the more complex *joint order batching and picker routing problem* (Wahlen and Gschwind, 2023) as a subproblem.

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